

The Sum of an Arithmetic Series

1. Determine the sum of each arithmetic series.

a) $3 + 12 + 21 + 30 + 39 + 48$ $n=6$

$$S_6 = \frac{6}{2}(3 + 48)$$

$$= 3(51) = 153$$

b) $19 + 31 + 43 + 55 + 67 + 79 + 91$ $n=7$

$$S_7 = \frac{7}{2}(19 + 91)$$

$$= 3.5(110)$$

$$= 385$$

c) $6 + 13 + 20 + 27 + 34 + 41 + 48 + 55$ $n=8$

$$S_8 = \frac{8}{2}(6 + 55)$$

$$= 4(61)$$

$$= 244$$

d) $25 + 31 + 37 + 43 + 49 + 55 + 61 + 67 + 73$ $n=9$

$$S_9 = \frac{9}{2}(25 + 73)$$

$$= 4.5(98)$$

$$= 441$$

2. For the arithmetic series $6 + 8 + 10 + 12 + \dots$

a) Determine the 20th term.

$$t_{20} = 6 + (20 - 1)(2)$$

$$= 6 + (19)(2)$$

$$= 6 + 38$$

$$t_{20} = 44$$

b) determine the sum of the first 20 terms.

$$S_{20} = \frac{20}{2}(6 + 44)$$

$$= 10(50)$$

$$= 500$$

3. Determine the sum of the first 10 terms of each arithmetic series.

a) $3 + 7 + 11 + \dots$

$$t_{10} = 3 + (10 - 1)4$$

$$= 3 + 36$$

$$= 39$$

$$S_{10} = \frac{10}{2}(3 + 39)$$

$$= 5(42)$$

$$S_{10} = 210$$

b) $5 + 11.5 + 18 + \dots$

$$t_{10} = 5 + (10 - 1)(6.5)$$

$$= 5 + 58.5$$

$$= 63.5$$

$$S_{10} = \frac{10}{2}(5 + 63.5)$$

$$= 5(68.5)$$

$$S_{10} = 342.5$$

c) $2 + 8 + 14 + \dots$

$$t_{10} = 2 + (10 - 1)6$$

$$= 2 + 54$$

$$= 56$$

$$S_{10} = \frac{10}{2}(2 + 56)$$

$$= 5(58)$$

$$S_{10} = 290$$

d) $45 + 39 + 33 + \dots$

$$t_{10} = 45 + (10 - 1)(-6)$$

$$= 45 + 9(-6)$$

$$= 45 - 54 = -9$$

$$S_{10} = \frac{10}{2}(45 + -9)$$

$$= 5(36)$$

$$S_{10} = 180$$

e) $6 + 16.8 + 27.6 + \dots$

$$t_{10} = 6 + (10 - 1)(10.8)$$

$$= 6 + 9(10.8)$$

$$= 6 + 97.2$$

$$= 103.2$$

$$S_{10} = \frac{10}{2}(6 + 103.2)$$

$$= 5(109.2)$$

$$S_{10} = 546$$

f) $21 + 15.1 + 9.2 + \dots$

$$t_{10} = 21 + (10 - 1)(-5.9)$$

$$= 21 + 9(-5.9)$$

$$= 21 - 53.1$$

$$= -32.1$$

$$S_{10} = \frac{10}{2}(21 + -32.1)$$

$$= 5(-11.1)$$

$$= -55.5$$

4. Tasty Treats finds that its profit from the sale of ice cream increases by \$5 per week during the 15-week summer season. Suppose the profit for the first week is \$30. Determine the profit for the season.

$$S_{15} = \frac{15}{2}(30 + 100)$$

$$= 7.5(130)$$

$$S_{15} = 975$$

$$t_{15} = 30 + (15 - 1)5$$

$$= 30 + 14(5)$$

$$= 30 + 70$$

$$t_{15} = 100$$

5. For three summer months (12 weeks), Job A pays \$325 per month with a monthly raise of \$100. Job B pays \$50 per week with a weekly raise of \$10. Which is the better-paying job?

JOB A

$$S_3 = \frac{3}{2} (325 + 525)$$

$$= 1.5 (850)$$

$$= \$1275$$

JOB B

$$S_{12} = \frac{12}{2} (50 + 160)$$

$$= 6 (210)$$

$$= \$1260$$

$$t_n = 50 + (12-1)10$$

$$= 50 + 11(10)$$

$$= 50 + 110$$

$$= 160$$

JOB A BETTER

6. For three summer months (12 weeks), Job A pays \$400 per month with a monthly raise of \$20. Job B pays \$100 per week with a weekly raise of \$5. Do the jobs pay the same total amount over the summer, or does one pay more than the other? Explain your answer.

JOB A

$$S_3 = \frac{3}{2} (400 + 440)$$

$$= 1.5 (840)$$

$$= \$1260$$

JOB B

$$S_{12} = \frac{12}{2} (100 + 155)$$

$$= 6 (255)$$

$$= \$1530$$

$$t_{12} = 100 + (12-1)5$$

$$= 100 + 11(5)$$

$$= 155$$

JOB B BETTER

7. In a supermarket, apple juice cans are stacked in a display arranged in layers. The numbers of cans in the layers form an arithmetic sequence. There are 48 cans in the bottom layer, and 20 cans in the top layer. There are 8 layers. How many cans are in the display?

$$S_8 = \frac{8}{2} (48 + 20)$$

$$= 4 (68)$$

$$= 272 \text{ cans. total}$$

8. Raji's annual salary is in a range from \$25 325 in the 1st year to \$34 445 in the 7th year.

- a) The salary range is an arithmetic sequence with seven terms. Determine the raise Raji can expect each year.

$$t_7 = a + (n-1)d$$

$$34,445 = 25,325 + (7-1)d$$

$$-25,325 \quad -25,325$$

$$9,120 = 6d$$

$$\frac{9,120}{6} = d$$

$$1,520 = d$$

- b) What is her salary in the fourth year?

$$t_4 = a + (n-1)d$$

$$= 25,325 + (4-1)(1,520)$$

$$= 25,325 + 3(1,520)$$

$$= 25,325 + 4,560$$

$$= \$29,885$$

c) In which year does her salary exceed \$30 000 for the first time?

$$t_n = a + (n-1)d$$

$$30,000 = 25,325 + (n-1)(1520)$$

$$-25,325 \quad -25,325$$

$$4,675 = 1520(n-1)$$

$$\rightarrow 3.07 = n-1$$

$$4.07 = n$$

d) What is the total amount Raji will earn in the seven years?

$$S_7 = \frac{7}{2} (25,325 + 34,445)$$

$$= 3.5 (59,770)$$

$$= 209,195$$

∴ 5 years

9. A pile of bricks is arranged in rows. The number of bricks in the rows form an arithmetic sequence. There are 35 bricks in the 4th row and 20 bricks in the 9th row.

a) How many bricks are in the first row?

9th row 20 $d = \frac{20-35}{9-4}$

4th row 35 $= \frac{-15}{5} = -3$

$$t_n = a + (n-1)d$$

$$35 = a + (4-1)(-3)$$

$$35 = a + 3(-3)$$

$$35 = a - 9$$

$$44 = a$$

b) How many rows of bricks are there?

$$t_n = a + (n-1)d$$

$$= 44 + (n-1)(-3)$$

$$= 44 - 3n + 3$$

$$= 47 - 3n$$

$$\rightarrow -46 = -3n$$

$$15.3 = n$$

There are 15 rows

c) How many bricks are in the pile?

$$S_{15} = \frac{15}{2} (44 + 2)$$

$$= 7.5 (46)$$

$$= 345$$

$$t_{15} = 44 + (15-1)(-3)$$

$$= 44 + 14(-3)$$

$$= 44 - 42$$

$$= 2$$

10. Determine the sum of each arithmetic series.

i) $2 + 7 + 12 + \dots + 62$

$$S_{13} = \frac{13}{2} (2 + 62)$$

$$= 6.5 (64)$$

$$= 416$$

$$62 = 2 + (n-1)5$$

$$60 = 5n - 5$$

$$65 = 5n$$

$$13 = n$$

ii) $4 + 11 + 18 + \dots + 88$

$$S = \frac{13}{2} (4 + 88)$$

$$= 6.5 (92)$$

$$= 598$$

$$88 = 4 + (n-1)7$$

$$84 = 7n - 7$$

$$91 = 7n$$

$$13 = n$$

iii) $3 + 5.5 + 8 + \dots + 133$

$$S_{53} = \frac{53}{2} (3 + 133)$$

$$= 26.5 (136)$$

$$= 3604$$

$$133 = 3 + (n-1)(2.5)$$

$$130 = 2.5n - 2.5$$

$$132.5 = 2.5n$$

$$53 = n$$

iv) $20 + 14 + 8 + \dots + (-40)$

$$S_{11} = \frac{11}{2} (20 + (-40))$$

$$= 5.5 (-20)$$

$$= -110$$

$$-40 = 20 + (n-1)(-6)$$

$$-60 = -6n + 6$$

$$-66 = -6n$$

$$11 = n$$

11. This sentence is called a "snowball sentence."

I do not know where family doctors acquired perplexing handwriting; nevertheless, extraordinary pharmaceutical intellectuality, counterbalancing indecipherability, transcendentalizes intercommunications' incomprehensibility.

a) Why is the name "snowball sentence" appropriate?

Each word in the sentence is one longer than previous word letter

b) How many letters are in this snowball sentence?

$$S_n = \frac{n(n+1)}{2}$$

$$= 10(21) = 210$$

$$20 = 1 + (n-1)$$

$$19 = n - 1$$

$$20 = n$$

12. In the popular TV quiz show "Jeopardy!", a contestant gives each response as a question to a clue hidden behind a panel that shows an amount of money. When contestant's response is correct, the contestant wins the money. Answer the questions below in as many different ways as you can.

a) In "Jeopardy!" (below), what is the total amount of money shown?

Word Origins	Oceans	Science	Movies	Modern Poetry	This & That
\$100	\$100	\$100	\$100	\$100	\$100
\$200	\$200	\$200	\$200	\$200	\$200
\$300	\$300	\$300	\$300	\$300	\$300
\$400	\$400	\$400	\$400	\$400	\$400
\$500	\$500	\$500	\$500	\$500	\$500

$$S_n = \frac{n}{2} (100 + 500)$$

$$= 2.5(600)$$

$$= 1500$$

$$1500 \times 6 = \$9,000$$

b) In "Double Jeopardy!" (below), what is the total amount shown?

Food	TV Quiz Show	Sports	Math	Drama	Odds & Ends
\$200	\$200	\$200	\$200	\$200	\$200
\$400	\$400	\$400	\$400	\$400	\$400
\$600	\$600	\$600	\$600	\$600	\$600
\$800	\$800	\$800	\$800	\$800	\$800
\$1000	\$1000	\$1000	\$1000	\$1000	\$1000

$$S_n = \frac{n}{2} (200 + 1000)$$

$$= 2.5(1200)$$

$$= 3000$$

$$3000 \times 6 = \$18,000$$

Answers:

1a) 153 b) 385 c) 244 d) 441 2a) 44 b) 500 3a) 210 b) 342.5 c) 290 d) 180 e) 546 f) -55.5

4) \$975

5) Job A

6) Job B pays more. Explanations may vary.

7) 272

8a) 1520 b) 2485 c) 544 d) \$209 195

9a) 44 b) 15 c) 345

10i) 416 ii) 598 iii) 3604 iv) -110

11a) Answers may vary. b) 210

12a) \$9000 b) \$18 000