GEOMETRY 9

Focus on Reasoning

TEACHER MATERIALS

Zoë Wakelin

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PAGE	
	TEACHER PAGES
T3	Overview of the content of Geometry 9.
T5	The individual concepts to be reviewed or taught before the student worksheets are assigned. Some optional examples and teaching ideas are included in these pages.
	STUDENT WORKSHEETS (to be duplicated for each student)
S2	Handy reference sheets for the students showing the concepts learned in Math 8. These sheets could be duplicated in a bright colour so that students can find them easily in their binders.
S4	The prescribed curriculum content.
S29	A set of problems involving geometry and other strands. These can be used throughout the year as students learn the required skills.
S31	Optional enrichment units. Because there is no indication on the worksheets that these are enrichment units, teachers may choose to use them as part of the regular course.

NOTE:

GEOMETRIC PROPERTIES

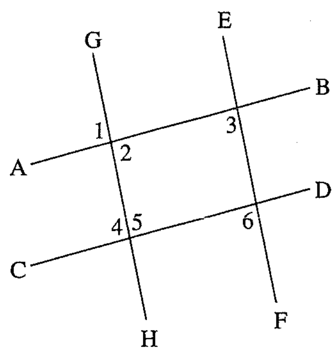
The emphasis in the geometry strand is on the discovery and use of properties rather than on the formal proof of theorems.

Content of Geometry 9

(Approx. 15% of Math 9)

Parallel Lines

Lines are parallel if
— alternate interior ∠s are equal
— corresponding ∠s are equal
— interior ∠s on the same side of the transversal are supplementary



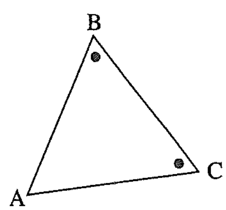
If $\angle 5 = \angle 6$, then GH || EF — alternate interior ∠s 5 and 6 are equal

If $\angle 1 = \angle 4$, then AB || CD — corresponding ∠s 1 and 4 are equal

If $\angle 2 + \angle 3 = 180°$, then GH || EF — interior ∠s on the same side of the transversal AB are supplementary

Congruent Sides

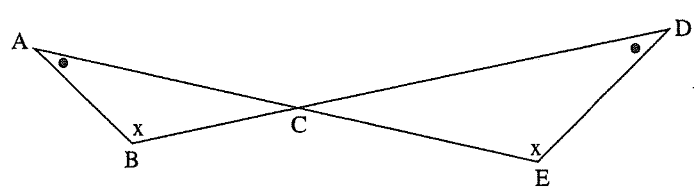
2 sides of a triangle are congruent if
— the ∠s opposite the sides are equal



If $\angle B = \angle C$, then AB = AC — sides opposite equal ∠s are equal

Similar Figures

2 figures are similar if
— corresponding ∠s are equal
— corresponding sides are in proportion



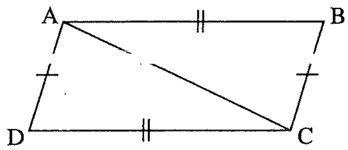
$\Delta ABC \sim \Delta DEC$ AAA

$\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$ corresponding sides of similar figures are in proportion

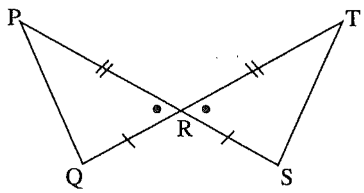
Congruent Triangles

Congruent triangles can be determined by

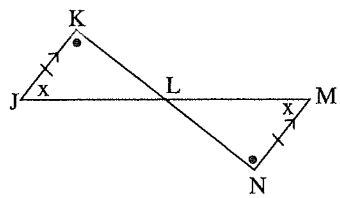
SSS	3 sides
SAS	2 sides and the contained angle
ASA	2 ∠s and the contained side



Δ ABC ≅ Δ CDA (SSS)



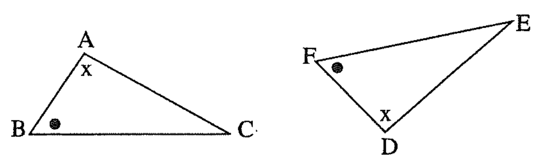
Δ PRQ ≅ Δ TRS (SAS)



Δ JKL ≅ Δ MNL (ASA)

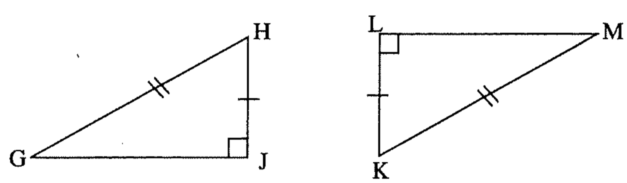
Note:

1. If 2 ∠s of one Δ are equal to 2 ∠s of another Δ, then the 3rd ∠s of each Δ will be equal. (∠ sum of Δ = 180°)



∠C = ∠E 3rd ∠s of Δs are equal

2. If 2 sides of a right Δ are equal to 2 corresponding sides of another right Δ, then the 3rd sides of each Δ will be equal. (Property of Pythagoras)



GJ = LM Property of Pythagoras

ANGLES, PARALLEL LINES

student worksheets pp. S4 to S6

I.L.O. 9.17, 9.18c

REVIEW

Angle properties of intersecting and parallel lines.
The emphasis in *Geometry 9* is on **writing reasons** for all statements.

Intersecting lines
Complementary angles add to 90°.
Supplementary angles add to 180°.
Angles on a line add to 180°.
Angles at a point add to 360°.
Congruent (equal) angles have the same measure.
Vertically opposite angles are equal.

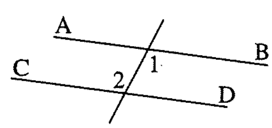
Parallel lines and transversals
If two lines are parallel and cut by a transversal, then
1. alternate interior angles are equal,
2. corresponding angles are equal,
3. interior angles on the same side of the transversal are supplementary.

Refer to:
GEOMETRY YOU SHOULD KNOW FROM MATH 8 (S2, S3)

NEW

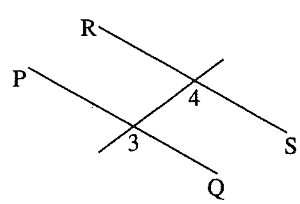
The converse of the properties of parallel lines above.

1. If alternate interior angles are equal, then the two lines are parallel.



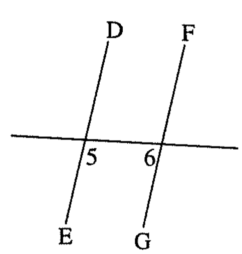
If $\angle 1 = \angle 2$, then AB || CD.

2. If corresponding angles are equal, then the two lines are parallel.



If $\angle 3 = \angle 4$, then PQ || RS.

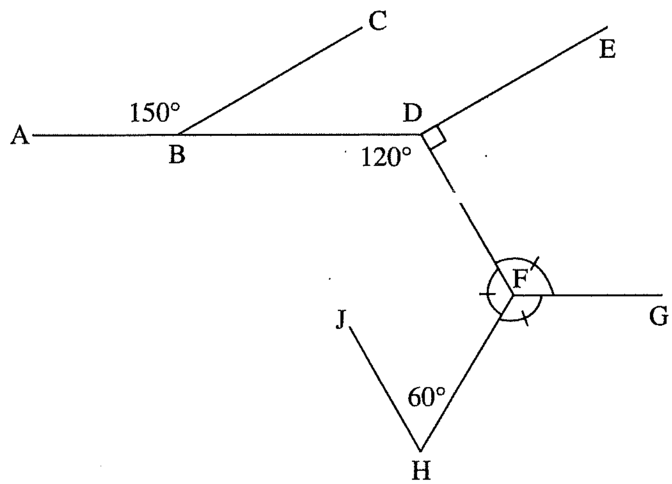
3. If the interior angles on the same side of the transversal are supplementary, then the two lines are parallel.



If $\angle 5 + \angle 6 = 180°$, then DE || FG.

INTERSECTING AND PARALLEL LINES: EXAMPLE

Identify all pairs of parallel segments in the diagram. State a reason for each answer.



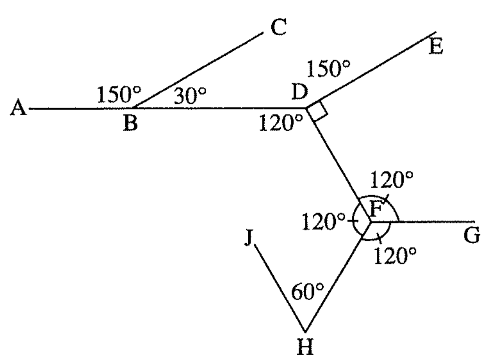
______ || ______ ______________________________

______ || ______ ______________________________

______ || ______ ______________________________

Solution

First calculate all the angles in the diagram.



BC || DE corresponding ∠s ABC, BDE are equal.

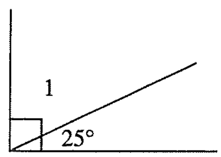
BD || FG alternate interior ∠s BDF, DFG are equal.

JH || DF interior ∠s DFH, JHF on the same side of the transversal are supplementary.

INTERSECTING LINES: ANSWERS

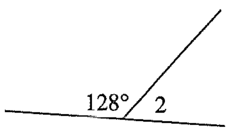
Find the measure of each required angle and give the reason for your answer.

1.



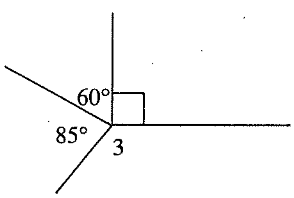
∠1 = *65°* *complementary ∠s*

2.



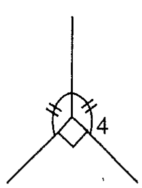
∠2 = *52°* *∠s on a line add to 180°*

3.



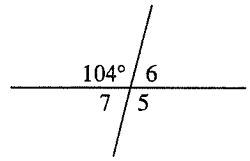
∠3 = ~~*135°*~~ 125° *∠s at a point add to 360°*

4.



∠4 = *135°* *∠s at a point add to 360°*

5.

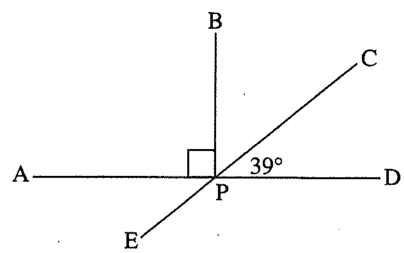


∠5 = *104°* *vertically opposite ∠s*

∠6 = *76°* *∠s on a line add to 180°*

∠7 = *76°* *vertically opposite ∠s or ∠s on a line add to 180°*

6.

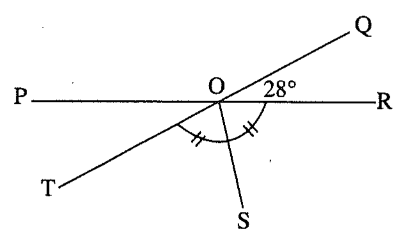


∠BPD = *90°* *∠s on a line add to 180°*

∠BPC = *51°* *complementary ∠s*

∠APE = *39°* *vert opp ∠s*

7.



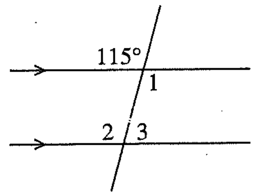
∠POT = *28°* *vert opp ∠s*

∠POQ = *152°* *∠s on a line add to 180°*

∠ROT = *152°* *vert opp ∠s*

∠ROS = *76°* *half of 152°*

8.

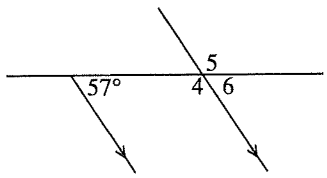


$\angle 1 =$ *115°* *vert opp ∠s*

$\angle 2 =$ *115°* *alternate interior ∠s*

$\angle 3 =$ *65°* *∠s on a line add to 180°*

9.

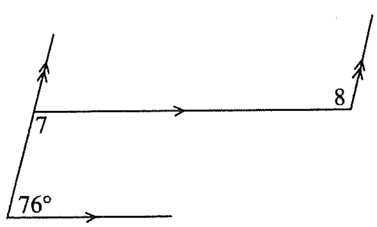


$\angle 4 =$ *123°* *int ∠s on same side of trans*

$\angle 5 =$ *123°* *vert opp ∠s*

$\angle 6 =$ *57°* *corresponding ∠s or ∠s on a line add to 180°*

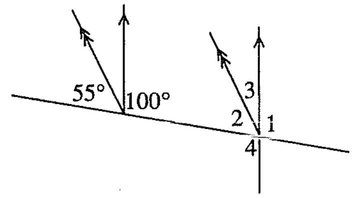
10.



$\angle 7 =$ *104°* *interior ∠s on same side of trans*

$\angle 8 =$ *104°* *alt int ∠s*

11.



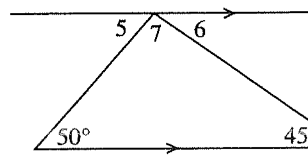
$\angle 1 =$ *100°* *corr ∠s*

$\angle 2 =$ *55°* *corr ∠s*

$\angle 3 =$ *25°* *∠s on a line add to 180°*

$\angle 4 =$ *100°* *vert opp to ∠1*

12.

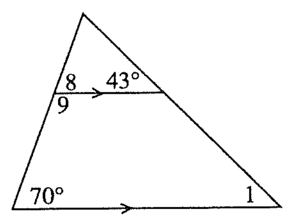


$\angle 5 =$ *50°* *alt int ∠s*

$\angle 6 =$ *45°* *alt int ∠s*

$\angle 7 =$ *85°* *∠s on a line add to 180°*

13.

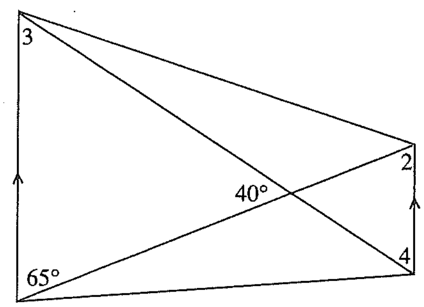


$\angle 8 =$ *70°* *corr ∠s*

$\angle 9 =$ *110°* *∠s on a line add to 180°*

$\angle 1 =$ *43°* *corr ∠s*

14.

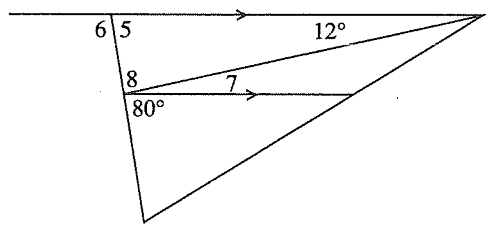


$\angle 2 =$ *65°* *alt int $\angle$s*

$\angle 3 =$ *75°* *$\angle$ sum of Δ = 180°*

$\angle 4 =$ *75°* *alt int $\angle$s*

15.



$\angle 5 =$ *80°* *corr $\angle$s*

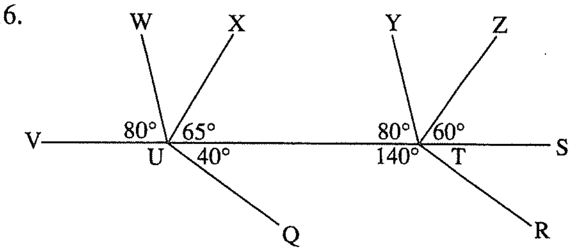
$\angle 6 =$ *100°* *$\angle$s on a line add to 180°*

$\angle 7 =$ *12°* *alt int $\angle$s*

$\angle 8 =$ *88°* *$\angle$s on a line add to 180°*

Name 2 pairs of parallel segments in each figure. State the reason for your answer.

16.

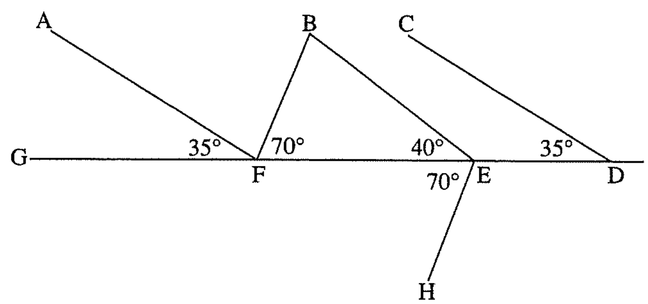


UW || *TY* *corr $\angle$s 80° are =*

QU || RT *int $\angle$s on same side of trans add to 180°*

(40° + 140° = 180°)

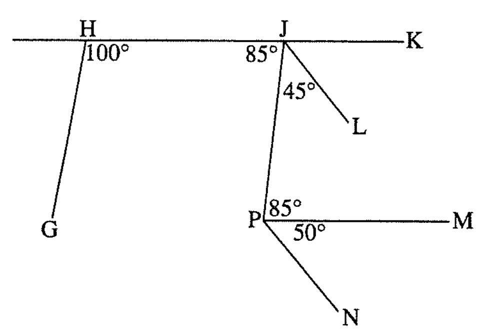
17.



AF || CD *corr $\angle$s 35° are =*

BF || HE *alt int $\angle$s 70° are =*

18.



JH || PM *alt int $\angle$s 85° are =*

JL || PN *int $\angle$s on same side of trans add to 180°*

[45° + (85° + 50°) = 180°]

I.L.O. 9.17, 9.18c

REVIEW

Properties of triangles.
The emphasis in *Geometry 9* is on **writing reasons** for all statements.

Triangle properties

Angle sum of a triangle is 180°.

Isosceles triangle
– 2 sides equal
– the angles opposite the equal sides are equal

Equilateral triangle
– 3 sides equal
– each angle is 60°

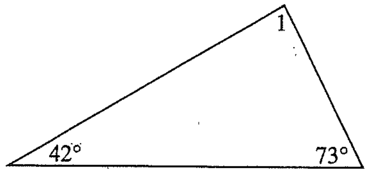
Right triangle
– 1 right triangle
– side opposite the right angle is the hypotenuse
– property of Pythagoras, $a^2 + b^2 = c^2$

Refer to:
GEOMETRY YOU SHOULD KNOW FROM MATH 8 (S2, S3)

TRIANGLES: ANSWERS

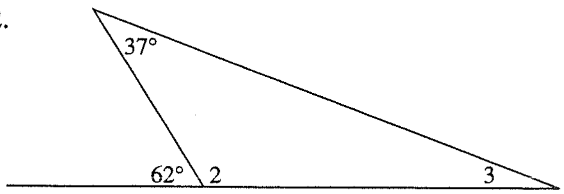
Find the measure of each required angle and give the reason for your answer.

1.



$\angle 1 =$ *65°* — *$\angle$ sum of $\Delta = 180°$*

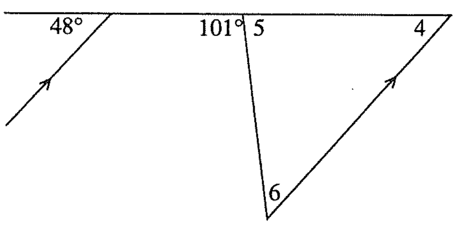
2.



$\angle 2 =$ *118°* — *$\angle$s on a line add to 180°*

$\angle 3 =$ *25°* — *$\angle$ sum of $\Delta = 180°$*

3.

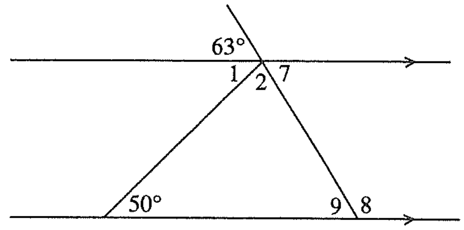


$\angle 4 =$ *48°* — *corr $\angle$s*

$\angle 5 =$ *79°* — *$\angle$s on a line add to 180°*

$\angle 6 =$ *53°* — *$\angle$ sum of $\Delta = 180°$*

4.



$\angle 7 =$ *63°* — *vert opp $\angle$s*

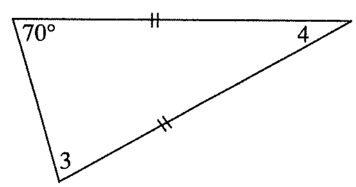
$\angle 8 =$ *117°* — *int $\angle$s on same side of trans*

$\angle 9 =$ *63°* — *alt int $\angle$s*

$\angle 1 =$ *50°* — *alt int $\angle$s*

$\angle 2 =$ *67°* — *$\angle$ sum of $\Delta = 180°$*

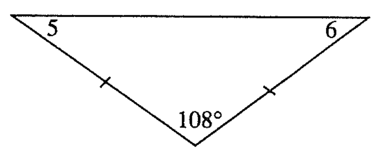
5.



$\angle 3 =$ *70°* — *$\angle$s opposite = sides or isos Δ*

$\angle 4 =$ *40°* — *$\angle$ sum of Δ*

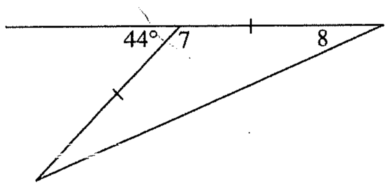
6.



$\angle 5 =$ *36°* — *isos Δ, $\angle$ sum of $\Delta = 180°$*

$\angle 6 =$ *36°* — *isos Δ*

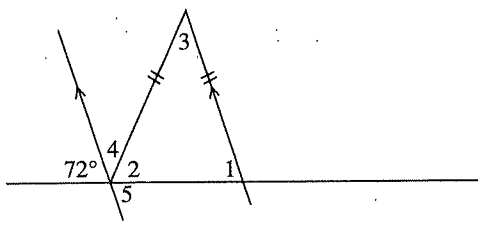
7.



$\angle 7 =$ *136°* *∠s on a line*

$\angle 8 =$ *22°* *isos Δ, ∠ sum of Δ*

8.



$\angle 1 =$ *72°* *corr ∠s*

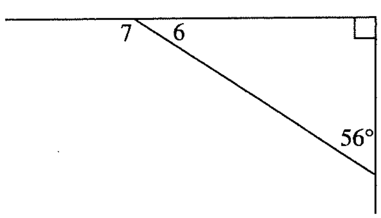
$\angle 2 =$ *72°* *isos Δ*

$\angle 3 =$ *36°* *∠ sum of Δ*

$\angle 4 =$ *36°* *alt int ∠ to ∠3*

$\angle 5 =$ *72°* *alt int ∠ to ∠1*

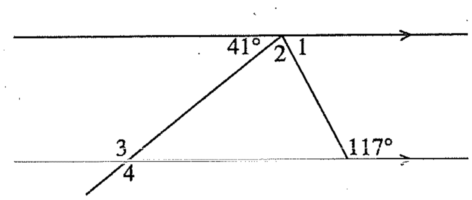
9.



$\angle 6 =$ *34°* *∠ sum of Δ*

$\angle 7 =$ *146°* *∠s on a line*

10.



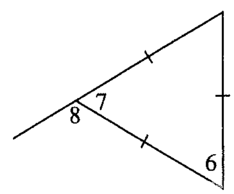
$\angle 1 =$ *63°* *int ∠s on same side of trans*

$\angle 2 =$ *76°* *∠s on a line*

$\angle 3 =$ *139°* *int ∠s on same side of trans*

$\angle 4 =$ *139°* *vert opp ∠s*

11.

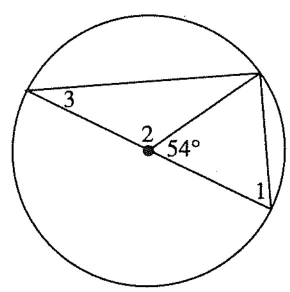


$\angle 6 =$ *60°* *equilateral Δ*

$\angle 7 =$ *60°* *equilateral Δ*

$\angle 8 =$ *120°* *∠s on a line*

12.

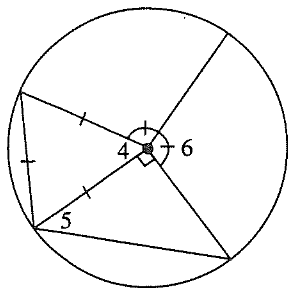


$\angle 1 =$ *63°* *isos Δ, radii are =*

$\angle 2 =$ *126°* *∠s on line*

$\angle 3 =$ *27°* *isos Δ, radii are =*

13.

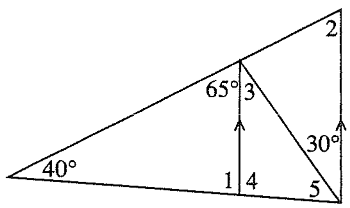


∠4 = *60°* *equilateral Δ*

∠5 = *45°* *isos Δ*

∠6 = *105°* *∠s at a point add to 360°*

14.



∠1 = *75°* *∠ sum of Δ*

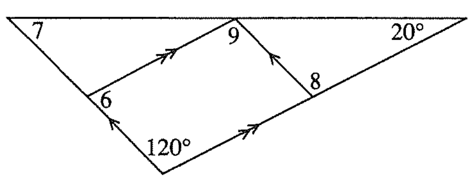
∠2 = *65°* *corr ∠s*

∠3 = *30°* *alt int ∠s*

∠4 = *105°* *∠s on a line*

∠5 = *45°* *∠ sum of Δ*

15.



∠6 = *60°* *int ∠s on same side of trans*

∠7 = *40°* *∠ sum of Δ*

∠8 = *120°* *corr ∠s*

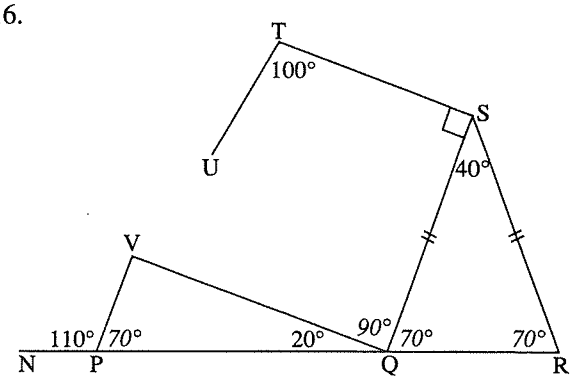
∠9 = *120°* *opp ∠s of | |gram are = or int*

∠s on same side of trans with

∠6 or alt int to ∠8

Name all the pairs of parallel segments in each figure. State the reason for your answer. *Reasons may vary from those stated.*

16.

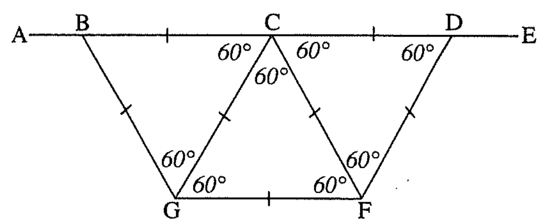


VP | | SQ *corr ∠s VPQ = SQR*

VQ | | ST *int ∠s on same side of trans*

(VQS + QST = 180°)

17.

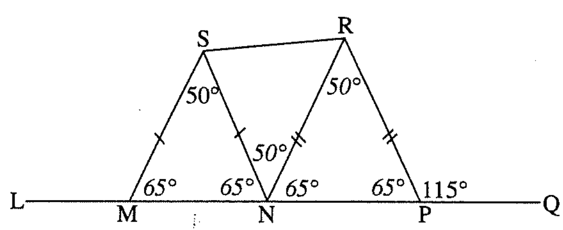


BD | | GF *alt int ∠s BCG = CGF*

BG | | CF *alt int ∠s BGC = GCF*

CG | | DF *alt int ∠s GCF = CFD*

18.



SM | | RN *corr ∠s SMN = RNP*

SN | | RP *corr ∠s SNM = RPN*

Note: *ΔSNR is not isosceles*

I.L.O. 9.17, 9.18c

REVIEW

Properties of quadrilaterals.
The emphasis in *Geometry 9* is on **writing reasons** for all statements.

Quadrilateral properties

Angle sum of a quadrilateral is 360°.

Trapezoid
– 1 pair of parallel sides
– interior angles on the same side of the transversal are supplementary

Parallelogram
– opposite sides are equal and parallel
– opposite angles are equal
– consecutive angles are supplementary
– diagonals bisect each other

Rectangle
– opposite sides are equal and parallel
– each angle is 90°
– diagonals are equal and bisect each other

Rhombus
– a parallelogram with 4 equal sides
– diagonals bisect each other at **right angles**
– diagonals bisect the angles of the rhombus

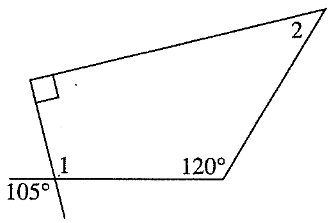
Square
– a rhombus with 4 right angles, or
– a rectangle with 4 equal sides

Refer to:
GEOMETRY YOU SHOULD KNOW FROM MATH 8 (S2, S3)

QUADRILATERALS: ANSWERS

Complete the following questions by naming the quadrilateral, finding the measures of angles and lengths, and giving reasons for your answers.

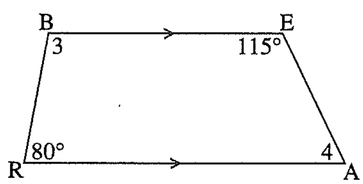
1.



$\angle 1 =$ *105°* *vert opp ∠s*

$\angle 2 =$ *45°* *∠ sum of quad = 360°*

2.

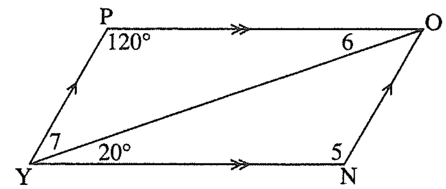


BEAR is a *trapezoid*

$\angle 3 =$ *100°* *int ∠s on same side of trans*

$\angle 4 =$ *65°* *int ∠s on same side of trans*

3.



PONY is a *parallelogram*

PY = *ON* *opp sides of | |gram are =*

$\angle 5 =$ *120°* *opp ∠s of | |gram are =*

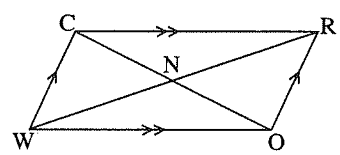
$\angle PYN =$ *60°* *int ∠s on same side of trans*

$\angle 6 =$ *20°* *alt int ∠s*

$\angle 7 =$ *40°* *adds with 20° to make ∠PYN or*

∠ sum of Δ

4.



CN = 2.7 cm
WR = 6.16 cm
$\angle CWO = 75°$

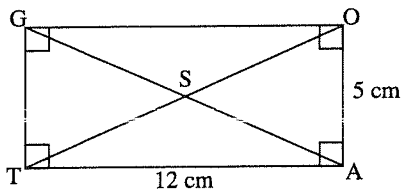
ON = *2.7* cm *diagonals of | |gram bisect*

WN = *3.08* cm *diagonals of | |gram bisect*

$\angle CRO =$ *75°* *opp ∠s of | |gram are =*

$\angle WOR =$ *105°* *int ∠s on same side of trans*

5.



GOAT is a *rectangle*

AG = TO *diags of rect are =*

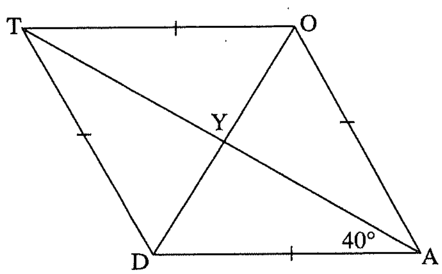
AG = *13* cm *Pythagoras*

GS = *6.5* cm *diags of rect bisect*

ΔGSO is *isos (GS = OS)*

ΔOAS is *isos (OS = AS)*

6.



TOAD is a *rhombus*

ΔDOT is *isos (OT = DT)*

∠DYA = *90°* *diags of rhombus bisect at 90°*

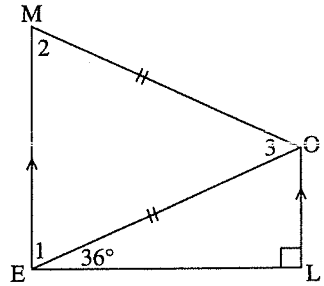
ΔDYT is *a right Δ*

∠DTA = *40°* *isos Δ (DT = DA)*

∠TDA = *100°* *∠ sum of Δ*

∠OTA = *40°* *diags of rhombus bisect the ∠s of the rhombus*

7.



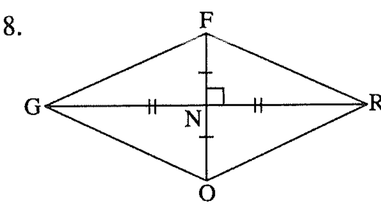
MOLE is a *trapezoid*

∠1 = *54°* *∠MEL + 90° = 180°*

∠2 = *54°* *isos Δ (MO = EO)*

∠3 = *72°* *∠ sum of Δ*

8.



FR = 10 cm
FN = 6 cm

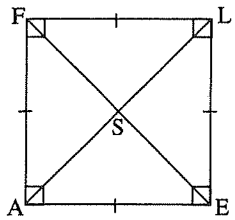
FROG is a *rhombus (diags bisect at 90°)*

FG = *10* cm *all sides of rhombus are =*

NR = *8* cm *Pythagoras*

FO = *12* cm *diags bisect*

9.



AE = 5 cm

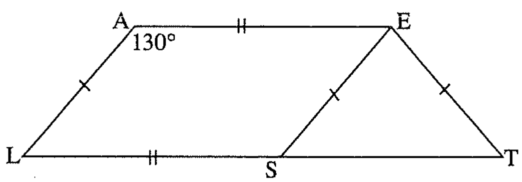
FLEA is a *square*

∠LSE = *90°* *diags bisect at 90°*

∠SEA = *45°* *isos Δ, ∠S = 90°*

FE = $\sqrt{50}$ cm *Pythagoras*

10.



SEAL is a *| |gram*

TEAL is a *trapezoid (isos trap)*

∠LSE = *130°* *opp ∠s of | |gram are =*

∠EST = *50°* *∠s on a line*

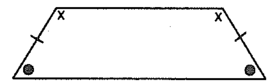
∠ETS = *50°* *isos Δ (ES = ET)*

∠ALT = *50°* *consecutive ∠s of | |gram add to 180°*

11. In question 10, TEAL is an **isosceles** trapezoid. What distinguishing properties does this shape have?

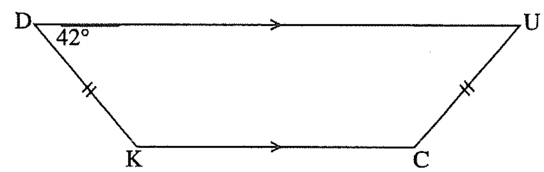
The non-parallel sides are equal.

2 pairs of consecutive angles are equal.



Diagonals are equal.

12.



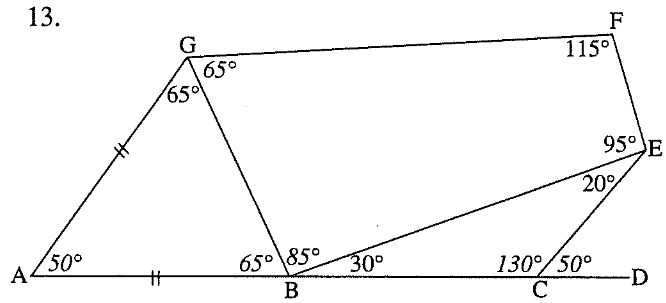
DUCK is a(n) *isosceles trapezoid*

∠DUC = *42°* *isos trap*

∠DKC = *138°* *int ∠s on same side of trans*

∠KCU = *138°* *isos trap*

13.



Name all the pairs of parallel segments in the diagram. State the reason for each answer.

AG | | CE *corr ∠s (GAB = ECD)*

GF | | AC *alt int ∠s (FGB = GBA)*

GB | | FE *int ∠s on same side of trans GBE, FEB, add to 180°*

14. Complete the following statements with a word or an expression to make each statement true.

a) If the diagonals of a parallelogram are equal, the parallelogram is a(n)

rectangle .

b) If the diagonals are perpendicular and *the diagonals*

bisect, the quadrilateral is a rhombus.

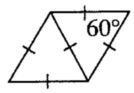
c) If the diagonals of a quadrilateral are equal, the

figure could be a *rectangle* or *square*

or *isosceles trapezoid* .

d) If one side of a rhombus is equal to the shorter diagonal, one of the angles of the rhombus

measures *60* degrees.



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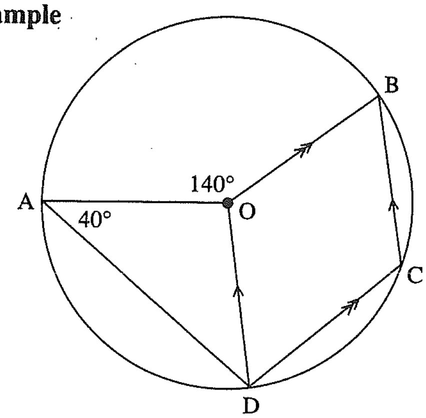
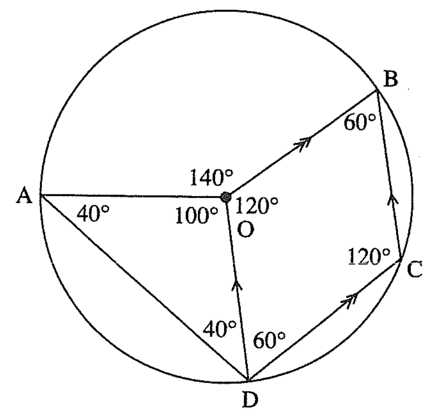
In the worksheets given so far, students have found the measures of several angles in a diagram by following the given sequence. In this section, however, although only a single angle is required, students will have to find other angles to reach the answer.

At first, students may be instructed to find all possible angles in a diagram until they reach the required one (questions 1 to 6).

Example

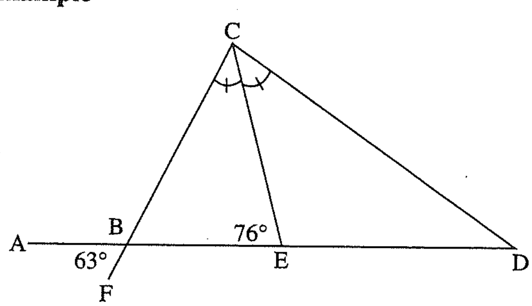


$\angle OBC = ?$

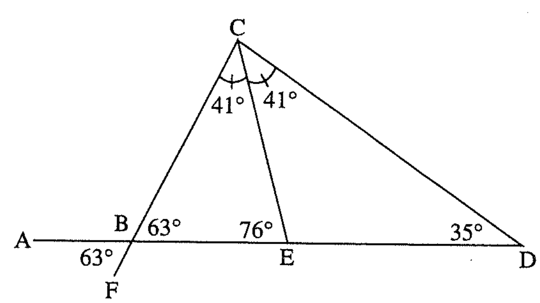


With practise, students will be able to list in sequence the minimum number of angles with reasons needed to reach the answer.

Example



$\angle CDE = ?$



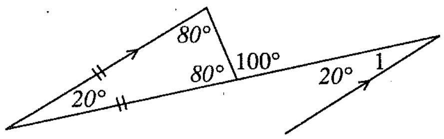
$\angle CBE = 63°$	vertically opposite $\angle$s
$\angle BCE = 41°$	$\angle$ sum of Δ BCE
$\angle BCD = 82°$	2 equal $\angle$s
$\angle CDE = 35°$	$\angle$ sum of ΔBCD

Note: There is insufficient space to list angles and reasons for questions 7 to 24. Students should work in their notebooks.

ANGLES: ANSWERS

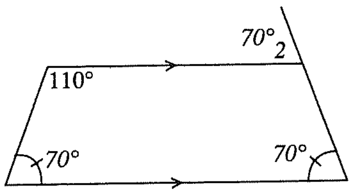
Find the measure of the required angle. Show clearly on the diagram any other angles you find.

1.



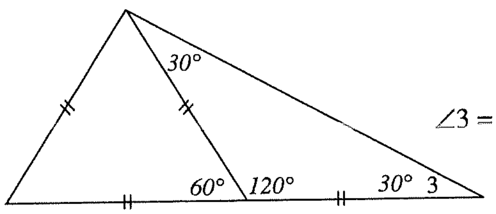
∠1 = 20°

2.



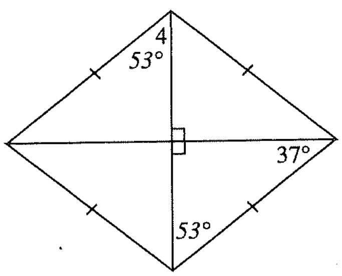
∠2 = 70°

3.



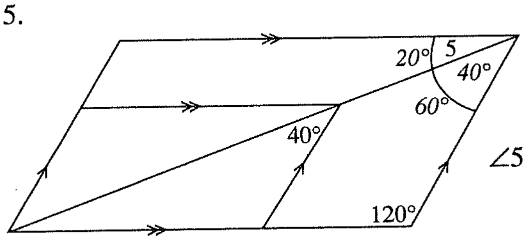
∠3 = 30°

4.



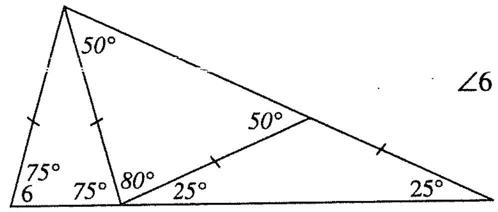
∠4 = 53°

5.



∠5 = 20°

6.

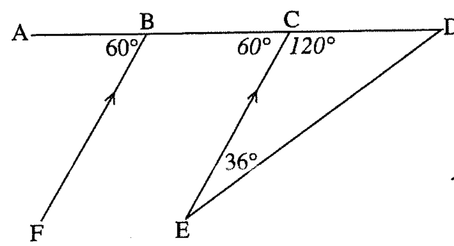


∠6 = 75°

Do the following questions in your notebook.

Find the measure of the required angle. List in sequence with reasons the angles you had to find to determine the required angle.

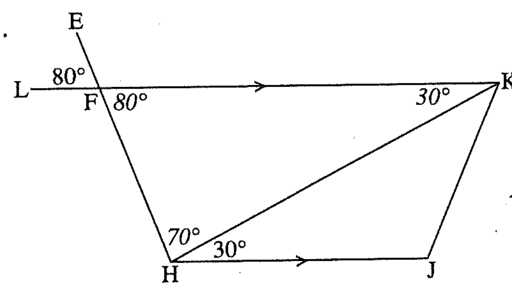
7.



24°

∠CDE =

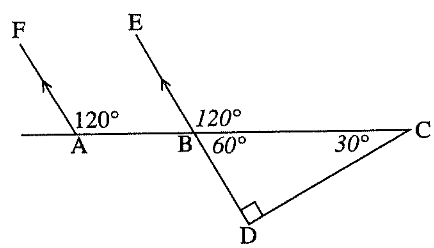
8.



70°

∠FHK =

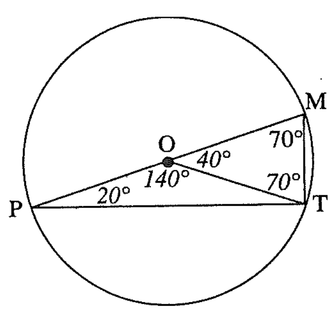
9.



30°

∠BCD =

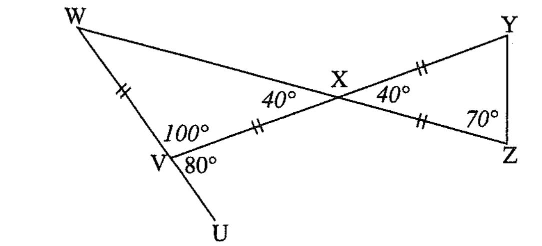
10.



20°

∠OPT =

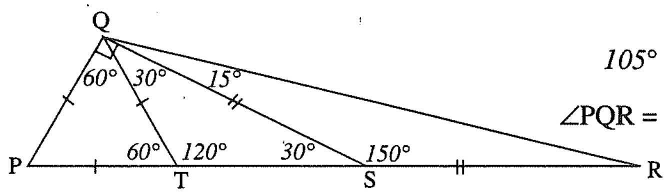
11.



70°

∠XZY =

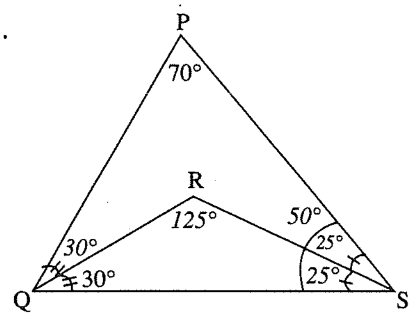
12.



105°

∠PQR =

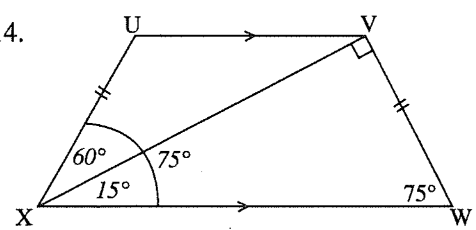
13.



125°

∠QRS =

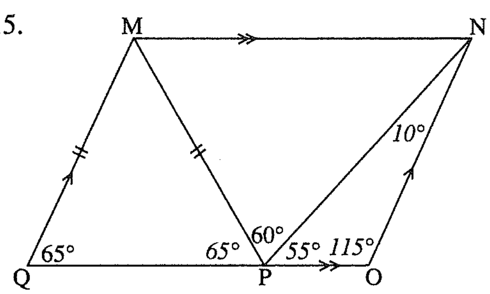
14.



60°

∠UXV =

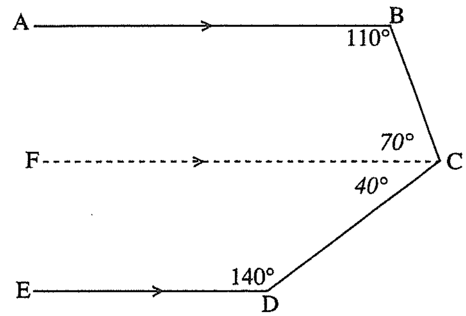
15.



10°

∠PNO =

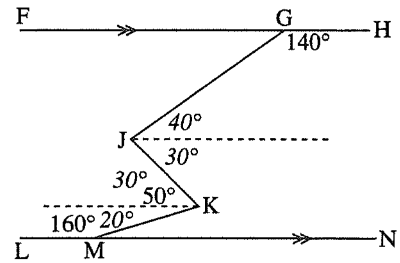
16.



110°

∠BCD =

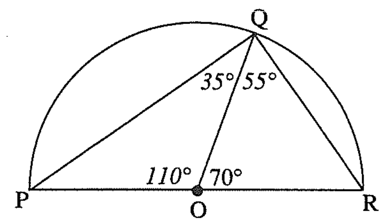
17.



70°

∠GJK =

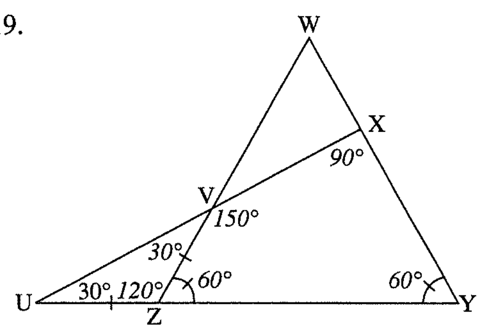
18.



90°

∠PQR =

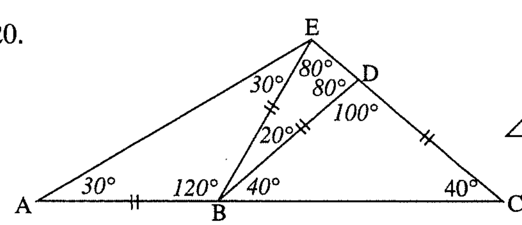
19.



90°

∠YXV =

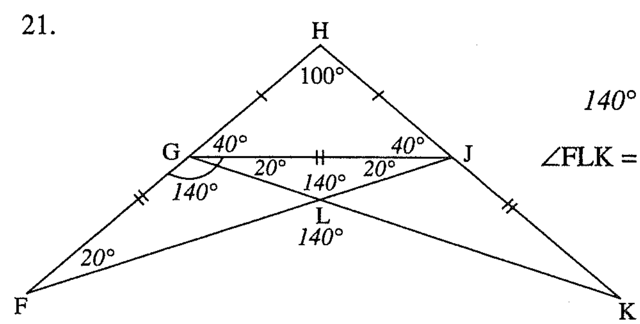
20.



30°

∠EAB =

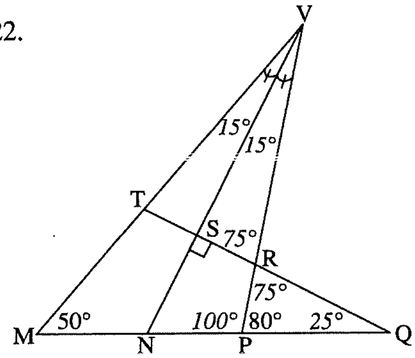
21.



140°

∠FLK =

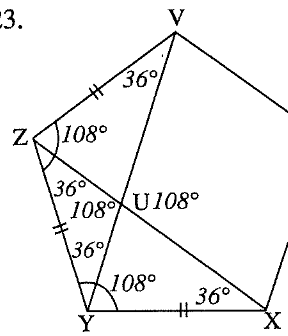
22.



∠PQR =

25°

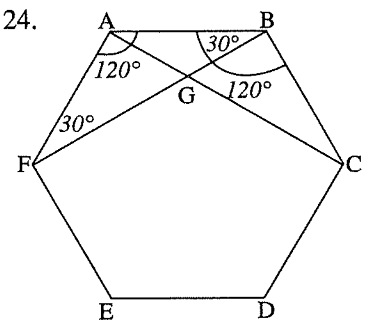
23.



VWXYZ is a regular pentagon. ∠VUX =

108°

24.



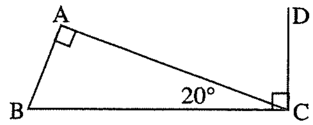
ABCDEF is a regular hexagon. ∠FBC =

90°

I.L.0. 9.18

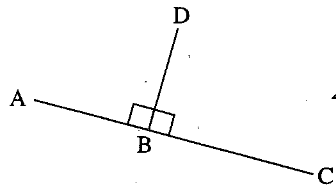
REVIEW

Angles are equal if they have the same measure,



$\angle B = 70°$	$\angle$ sum of Δ is 180°
$\angle ACD = 70°$	complementary $\angle$s
$\angle ACD = \angle B$	both equal 70°

If segments are perpendicular, they intersect at right angles.

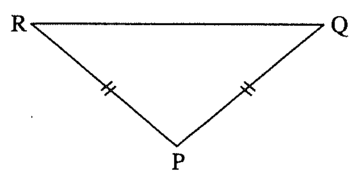


$BD \perp AC$	given
$\angle ABD = \angle DBC = 90°$	definition of $\perp$

Segments are parallel if
- alternate interior angles are equal,
- corresponding angles are equal,
- interior angles on the same side of the transversal are supplementary.

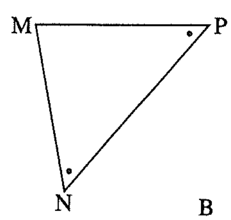
See page T5.

A triangle is isosceles if
- 2 sides are equal



$PQ = PR$	given
Δ PQR is isosceles	2 sides PQ and PR are equal

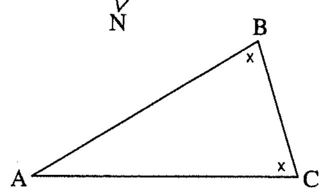
- 2 angles are equal



$\angle N = \angle M$	given
ΔMNP is isosceles	2 $\angle$s N and M are equal

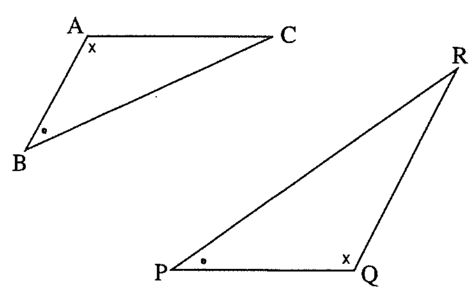
NEW

2 sides of a triangle are equal if the angles opposite the sides are equal.



$\angle B = \angle C$	given
$AB = AC$	sides opposite equal angles are equal

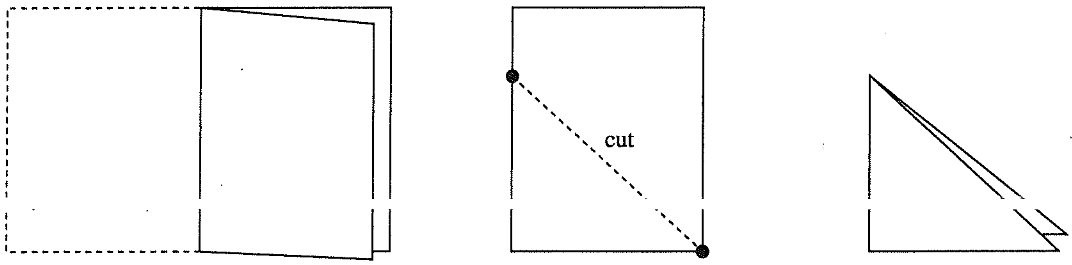
If 2 angles of one triangle equal 2 angles of another triangle, then the 3rd angles of each triangle are equal.



$\angle A = \angle Q$	given
$\angle B = \angle P$	given
$\angle C = \angle R$	3rd $\angle$s of the Δs are equal

TEACHING ACTIVITIES

Isosceles triangle



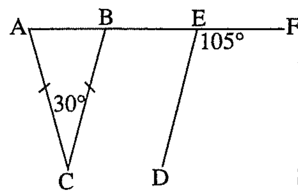
Have students fold a piece of paper in half and then cut along a line from any point on the fold to the corner. Explore the properties of the triangle that is formed.

– 2 sides are equal.
– 2 angles opposite the equal sides are equal.
– the fold line bisects the base.
– the fold line is perpendicular to the base.
– the fold line bisects the vertex angle.

GUIDED PROOFS: ANSWERS

Complete each of the following proofs.

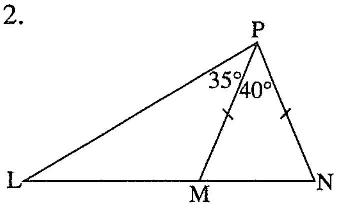
1.



Given: AC = BC, ∠ACB = 30°, ∠DEF = 105°
Show: BC | | ED

statement	reason
AC = BC	*given*
∠BAC = ∠ABC = *75°*	*∠s opp = sides (isos Δ)*
∠CBE = *105°*	*∠s on a line add to 180°*
∠CBE = ∠DEF	*both = 105°*
BC | | DE	*corr ∠s CBE, DEF are =*

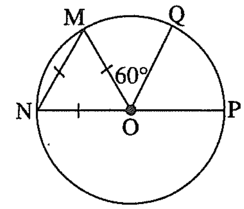
2.



Given: PN = PM, ∠LPM = 35°, ∠MPN = 40°
Show: LM = PN

statement	reason
PN = PM	*given*
∠PMN = ∠PNM = *70°*	*isos Δ*
∠LMP = *110°*	*∠s on line*
∠MLP = *35°*	*∠ sum of Δ*
∠MLP = ∠MPL	*both = 35°*
LM = PM	*sides opp = ∠s are =*
LM = PN	*both = PN*

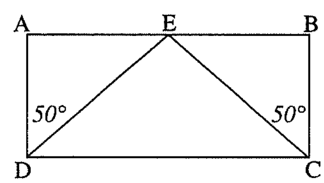
3.



Given: ΔMNO is equilateral, ∠MOQ = 60°
Show: MN | | OQ

statement	reason
ΔMNO is equilateral	*given*
∠NMO = *60°*	*equilateral Δ*
∠MOQ = *60°*	*given*
∠NMO = ∠MOQ	*both = 60°*
MN | | OQ	*alt int ∠s NMO, MOQ are =*

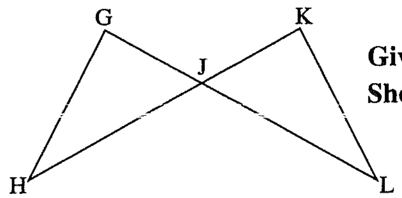
4.



Given: ABCD is a rectangle, ∠ADE = ∠BCE = 50°
Show: ED = EC

statement	reason
ABCD is a rectangle	*given*
∠ADC = ∠BCD = *90°*	*defn of rectangle*
∠ADE = ∠BCE = *50°*	*given*
∠EDC = *40°*	*90° – 50°*
∠ECD = *40°*	*90° – 50°*
∠EDC = ∠ECD	*both = 40°*
ED = EC	*sides opp = ∠s are =*

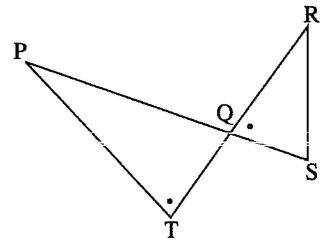
5.



Given: $\angle G = \angle K$
Show: $\angle H = \angle L$

statement	reason
$\angle G = \angle K$	*given*
$\angle GJH = \angle KJL$	*vert opp* $\angle$*s*
$\angle H = \angle L$	*3rd* $\angle$*s of* Δ*s are =*

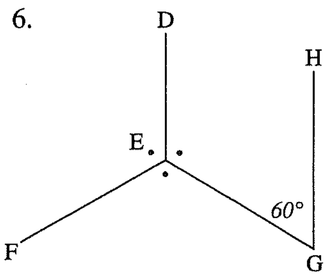
7.



Given: $\angle PTQ = \angle RQS$
Show: ΔPQT is isosceles

statement	reason
$\angle PTQ = \angle RQS$	*given*
$\angle PQT = \angle RQS$	*vert opp* $\angle$*s*
$\angle PTQ = \angle PQT$	*both =* $\angle RQS$
ΔPQT is isosceles	*2* $\angle$*s are =*

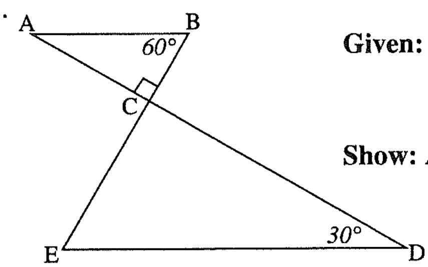
6.



Given:
$\angle DEF = \angle DEG = \angle FEG$,
$\angle EGH = 60°$
Show: DE || HG

statement	reason
$\angle DEF = \angle DEG = \angle FEG$	*given*
$\angle DEG =$ *120°*	$\angle$*s at a point add to 360°*
$\angle EGH =$ *60°*	*given*
$\angle DEG$ and $\angle DGH$ are supplementary	*both add to 180°*
DE || GH	*int* $\angle$*s on same side of trans DEG, DGH add to 180°*

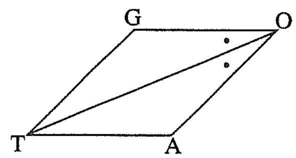
8.



Given: $BE \perp AD$,
$\angle ABC = 60°$,
$\angle CDE = 30°$
Show: AB || ED

statement	reason
$BE \perp AD$	*given*
$\angle ACB = \angle ECD =$ *90°*	*defn of* $\perp$
$\angle ABC =$ *60°*	*given*
$\angle CAB =$ *30°*	$\angle$ *sum of* Δ
$\angle CDE =$ *30°*	*given*
$\angle CAB = \angle CDE$	*both = 30°*
AB || ED	*alt int* $\angle$*s CAB, CDE are =*

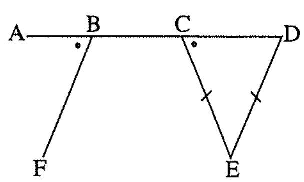
9.



Given: GOAT is a parallelogram, ∠GOT = ∠AOT
Show: GO = GT

statement	**reason**
∠GOT = ∠AOT	*given*
GT ∣∣ AO	*defn of ∣∣gram*
∠GTO = ∠AOT	*alt int angles*
∠GOT = ∠GTO	*both = ∠AOT*
GO = GT	*sides opp = ∠s are =*

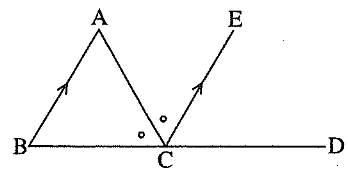
10.



Given: EC = ED, ∠ABF = ∠ECD
Show: BF ∣∣ DE

statement	**reason**
EC = ED	*given*
∠ECD = ∠EDC	*∠s opp = sides are =*
∠ECD = ∠ABF	*given*
∠EDC = ∠ABF	*both = ∠ECD*
BF ∣∣ DE	*corr ∠s EDC, ABF are =*

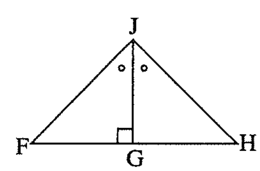
11.



Given: ∠ACB = ∠ACE, AB ∣∣ CE
Show: BA = BC

statement	**reason**
∠ACB = ∠ACE	*given*
AB ∣∣ CE	*given*
∠BAC = ∠ACE	*alt int ∠s*
∠ACB = ∠BAC	*both = ∠ACE*
BA = BC	*sides opp = ∠s are =*

12.



Given: JG ⊥ FH, ∠FJG = ∠HJG
Show: ΔJFH is isosceles

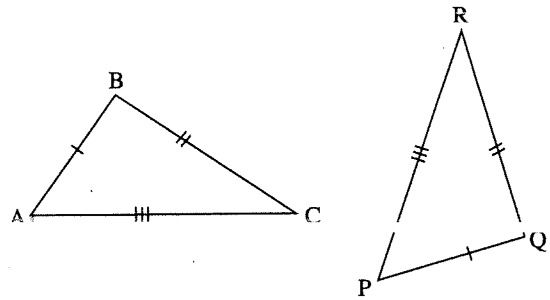
statement	**reason**
JG ⊥ FH	*given*
∠FGJ = ∠HGJ = *90°*	*defn of ⊥*
∠FJG = ∠HJG	*given*
∠JFG = ∠JHG	*3rd ∠s of Δs are =*
ΔJFH is isosceles	*2 ∠s are =*

I.L.O. 9.19

NEW

Triangles are congruent if

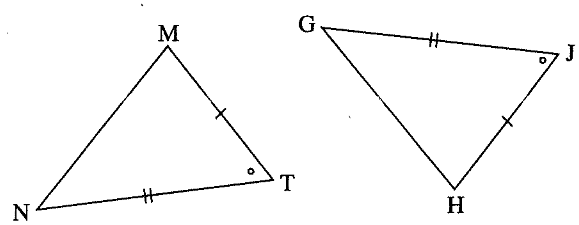
- 3 sides of one triangle equal 3 sides of the other triangle (SSS).



ΔABC ≅ ΔPQR SSS

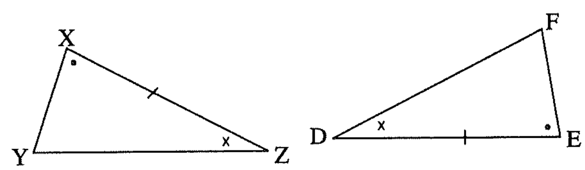
The order of the letters in naming the triangles indicates the correspondence of the vertices.

–2 sides and the contained angle of one triangle equal 2 sides and the contained angle of the other triangle (SAS).



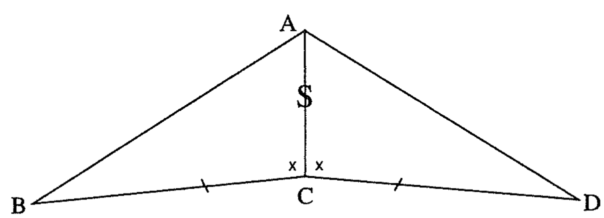
Δ MNT ≅ Δ HGJ SAS

– 2 angles and the contained side of one triangle equal 2 angles and the contained side of the other triangle (ASA).



Δ XYZ ≅ Δ EFD ASA

Triangles sharing a side

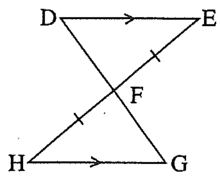


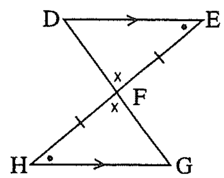
(S)	BC = CD	given
(A)	∠ACB = ∠ACD	given
(S)	AC = AC	same side
	Δ ABC ≅ Δ ADC	SAS

Deduced information

Students can use given information to deduce other facts about the triangles and can then use these facts to determine congruence. Encourage students to mark given and deduced information clearly on all diagrams.

Original diagram **Deduced information added**



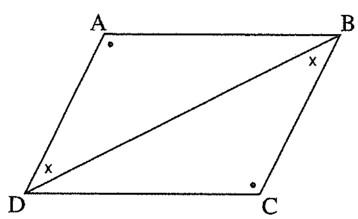


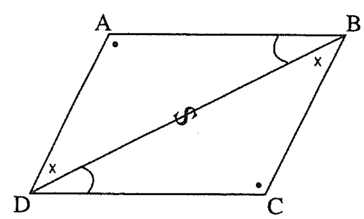
	DE | | HG	given
(A)	∠E = ∠H	alternate interior ∠s
(S)	EF = FH	given
(A)	∠DFE = ∠HGF	vertically opposite ∠s
	Δ DEF ≅ Δ GHF	ASA

> **Note:** Only SSS, SAS, and ASA congruence rules have been used throughout this material.

The **SAA rule** has not been used because the third angles of each triangle are equal. The ASA rule can then be used instead.

Original diagram **Deduced information added**

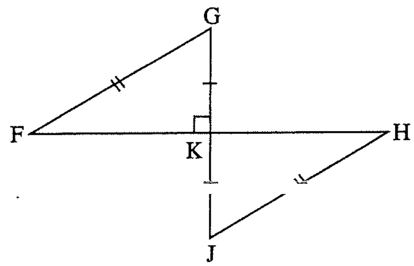




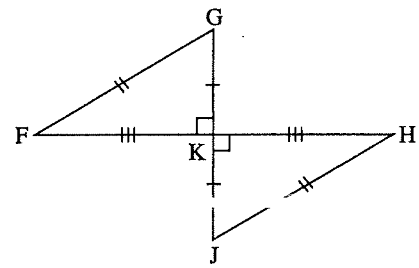
	∠A = ∠C	given
(A)	∠ADB = ∠CBD	given
(S)	DB = DB	same side
(A)	∠ABD = ∠CDB	3rd ∠s of Δs are equal
	Δ ABD ≅ Δ CDB	ASA

The **HL (Hypotenuse-Leg) rule** has not been used because the third sides of the right triangles are equal by the property of Pythagoras. SSS or SAS can then be used instead.

Original diagram



Deduced information added



	$\angle FKG = 90°$	given
	$\angle HKJ = 90°$	vertically opposite $\angle$s
(S)	FG = JH	given
(S)	GK = JK	given
(S)	FK = KH	Pythagoras
	Δ FGK $\cong$ Δ HJK	SSS (or SAS)

TEACHING ACTIVITIES

1. Have students draw triangles with the information given below. Determine which sets of information produce a unique triangle.

   a) Δ XYZ; XY = 6 cm, YZ = 8 cm, XZ = 11 cm.

   b) Δ ABC; AB = 10 cm, $\angle A = 35°$, $\angle B = 48°$.

   c) Δ FGH; FG = 3 cm, $\angle F = 64°$, FH = 8 cm.

   d) Δ MNP; $\angle M = 45°$, $\angle N = 100°$, NP = 7 cm.

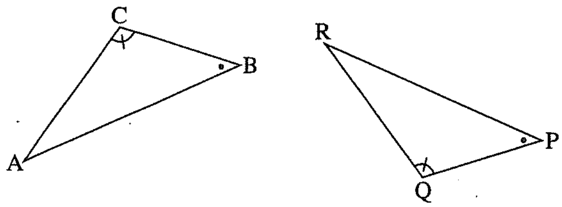
   e) Δ RST; ST = 5 cm, RT = 4 cm, $\angle S = 30°$.

   f) Δ UVW; $\angle U = 35°$, $\angle V = 105°$, $\angle W = 40°$.

CONGRUENT TRIANGLES: ANSWERS

1. Complete the congruence statement for each pair of triangles.

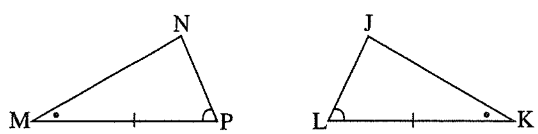
Example:



Δ ABC $\cong$ Δ RPQ

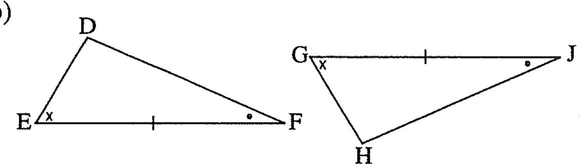
The order of the letters indicates the correspondence of the vertices.

a)



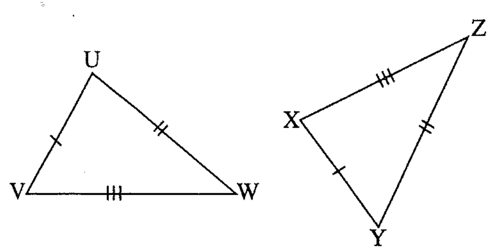
Δ MNP $\cong$ Δ *KJL*

b)



Δ DEF $\cong$ Δ *HGJ*

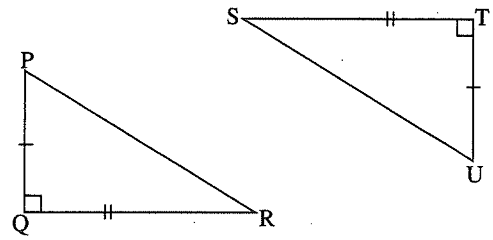
c)



Δ UVW $\cong$ Δ *YXZ*

($\angle$U and $\angle$Y are formed by segments with 2 marks and 1 mark.)

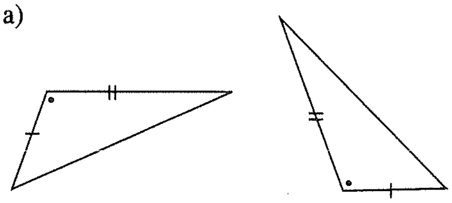
d)



ΔPQR $\cong$ Δ*UTS*

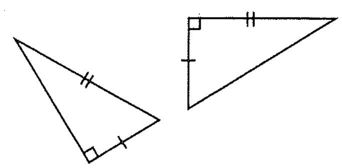
2. Are the following pairs of triangles congruent? If yes, name the congruency rule.

a)



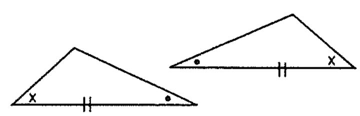
SAS

b)



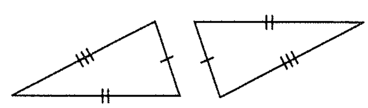
No

c)



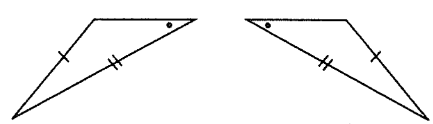
ASA

d)



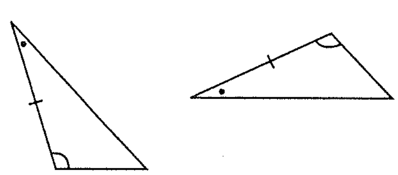
SSS

e)



No

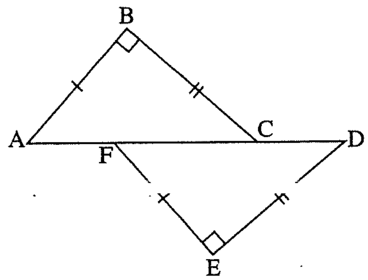
f)



ASA

3. Are the following pairs of triangles congruent? If yes, name the congruent triangles and the congruency rule, SSS, SAS, or ASA.

a)

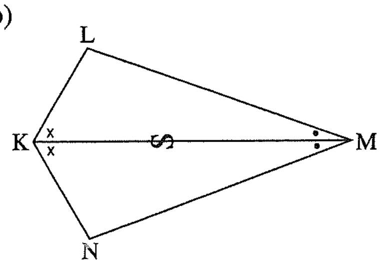


Yes/No *Yes*

$\Delta ABC \cong \Delta FED$

Rule *SAS*

b)

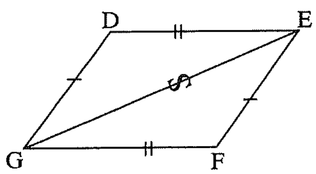


Yes

$\Delta KLM \cong \Delta KNM$

ASA

c)

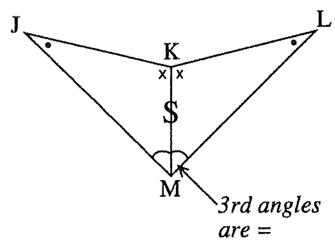


Yes

$\Delta DEG \cong \Delta FGE$

SSS

d)

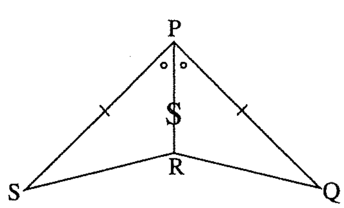


Yes

$\Delta JKM \cong \Delta LKM$

ASA

e)

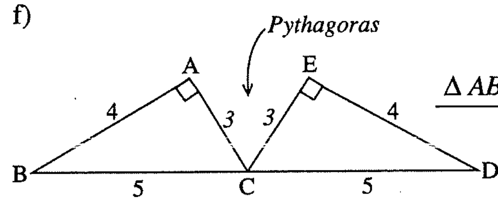


Yes

$\Delta PRS \cong \Delta PRQ$

SAS

f)



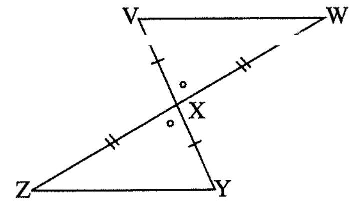
Yes

$\Delta ABC \cong \Delta EDC$

SAS

or SSS

g)

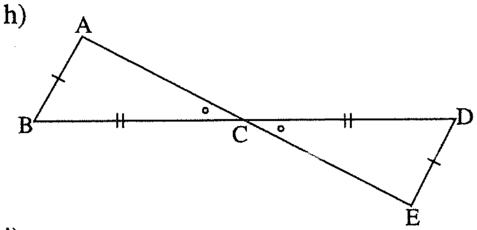


Yes

$\Delta VXW \cong \Delta YXZ$

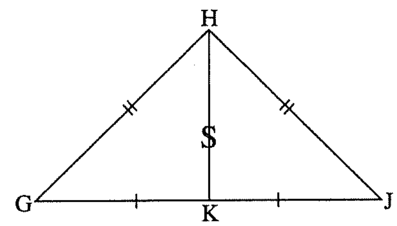
SAS

h)



No

i)

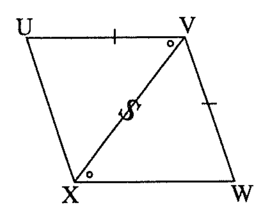


Yes

$\Delta HGK \cong \Delta HJK$

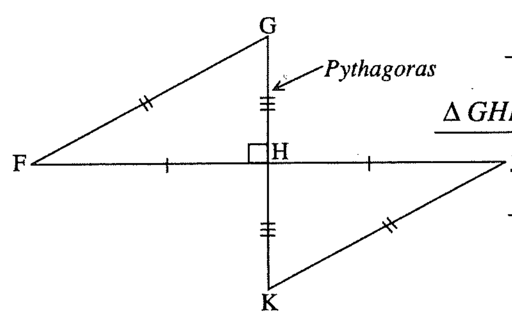
SSS

j)



No

k)



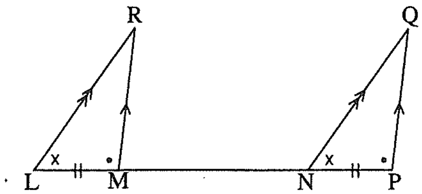
Yes

$\Delta GHF \cong \Delta KHJ$

SAS

or SSS

l)

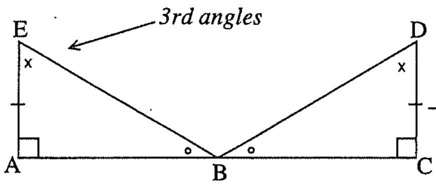


Yes

$\Delta RLM \cong QNP$

ASA

m)

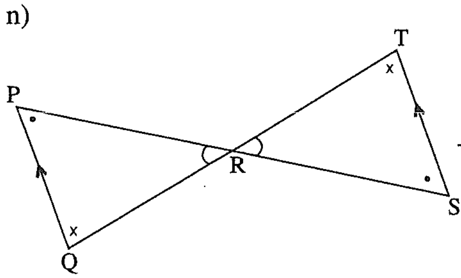


Yes

$\Delta EAB \cong \Delta DCB$

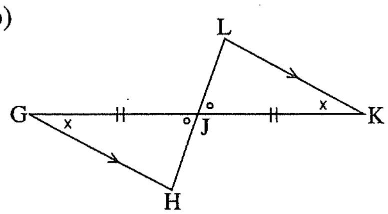
ASA

n)



No

o)

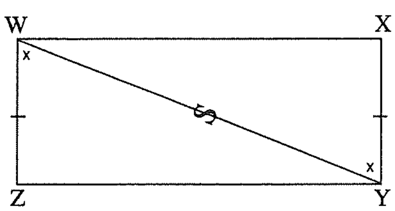


Yes

$\Delta GJH \cong \Delta KJL$

ASA

p)

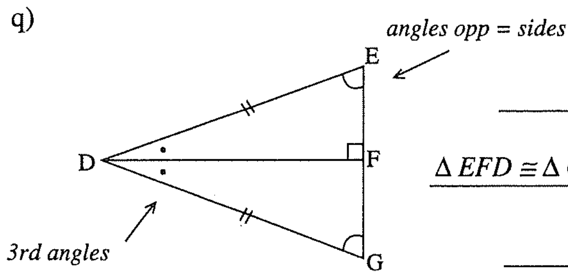


Yes

$\Delta WXY \cong \Delta YZW$

SAS

q)



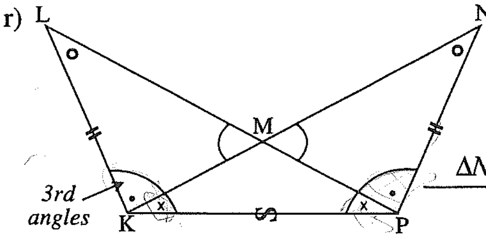
Yes

$\Delta EFD \cong \Delta GFD$

ASA

or SAS

r)



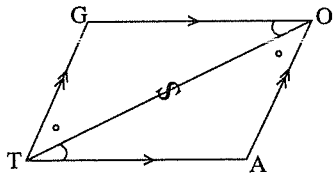
Yes

$\Delta NPK \cong \Delta LKP$

ASA or SAS

Complete each of the following proofs.

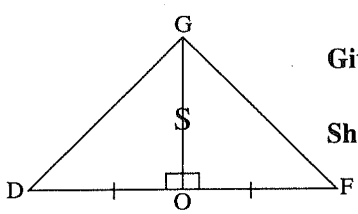
4.



Given: GO || TA, GT || OA
Show: $\Delta GOT \cong \Delta ATO$

statement	reason
GO || TA, GT || OA	*given*
$\angle GOT = \angle ATO$	*alt int $\angle$s*
OT = OT	*same side*
$\angle GTO = \angle AOT$	*alt int $\angle$s*
Δ GOT $\cong \Delta$ ATO	*ASA*

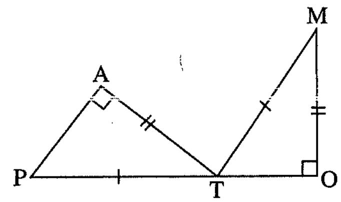
5.



Given: GO $\perp$ DF, DO = OF
Show: $\Delta DOG \cong \Delta FOG$

statement	reason
GO $\perp$ DF	*given*
$\angle DOG = \angle FOG = 90°$	*defn of $\perp$*
GO = GO	*same side*
DO = FO	*given*
Δ DOG $\cong \Delta$ FOG	*SAS*

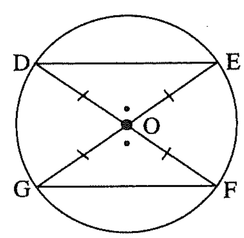
6.



Given: $\angle A, \angle O = 90°$, PT = TM, AT = MO
Show: $\Delta PAT \cong \Delta TOM$

statement	reason
$\angle A = \angle O = 90°$	*given*
PT = TM	*given*
AT = MO	*given*
PA = TO	*Pythagoras*
Δ PAT $\cong \Delta$ TOM	*SAS or SSS*

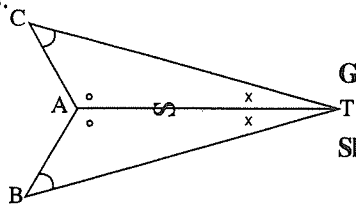
7.



Given: O is centre
Show: $\Delta DOE \cong \Delta FOG$

statement	reason
O is the centre	*given*
OD = OF	*radii are =*
OE = OG	*radii are =*
$\angle DOE = \angle FOG$	*vert opp $\angle$s*
Δ DOE $\cong \Delta$ FOG	*SAS*

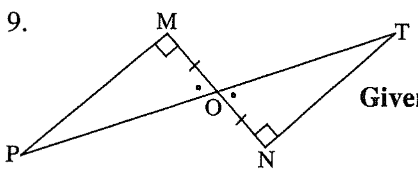
8.



Given: $\angle ACT = \angle ABT$, $\angle CTA = \angle BTA$
Show: $\Delta CAT \cong \Delta BAT$

statement	**reason**
$\angle ACT = \angle ABT$	*given*
$\angle CTA = \angle BTA$	*given*
$AT = AT$	*same side*
$\angle CAT = \angle BAT$	*3rd $\angle$s of Δ s are =*
$\Delta CAT \cong \Delta BAT$	*ASA*

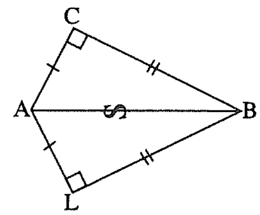
9.



Given: $MP \perp MN$, $TN \perp MN$, $MO = NO$
Show: $\Delta MOP \cong \Delta NOT$

statement	**reason**
$MP \perp MN$, $TN \perp MN$	*given*
$\angle OMP = \angle ONT = 90°$	*defn of $\perp$*
$ON = OM$	*given*
$\angle MOP = \angle NOT$	*vert opp $\angle$s*
$\Delta MOP \cong \Delta NOT$	*ASA*

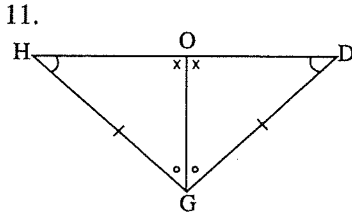
10.



Given: $\angle C = \angle L = 90°$, $AC = AL$
Show: $\Delta CAB \cong \Delta LAB$

statement	**reason**
$\angle C = \angle L = 90°$	*given*
$AC = AL$	*given*
$AB = AB$	*same side*
$CB = LB$	*Pythagoras*
$\Delta CAB \cong \Delta LAB$	*SAS or SSS*

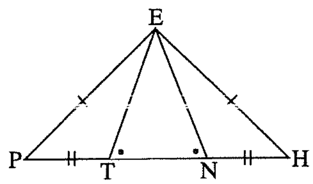
11.



Given: $HG = DG$, $\angle HOG = \angle DOG$
Show: $\Delta HOG \cong \Delta DOG$

statement	**reason**
$HG = DG$	*given*
$\angle GHO = \angle GDO$	*$\angle$s opp = sides are =*
$\angle HOG = \angle DOG$	*given*
$\angle OGH = \angle OGD$	*3rd $\angle$s of Δs are =*
$\Delta HOG \cong \Delta DOG$	*ASA*

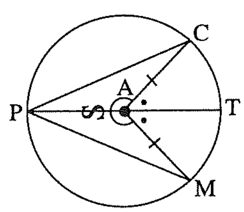
12.



Given: PE = EH,
PT = NH,
$\angle ETN = \angle ENT$
Show: Δ PET $\cong$ Δ HEN

statement	reason
$\angle ETN = \angle ENT$	*given*
ET = EN	*sides opp = $\angle$s are =*
PE = HE	*given*
PT = NH	*given*
Δ PET $\cong$ Δ HEN	*SSS*

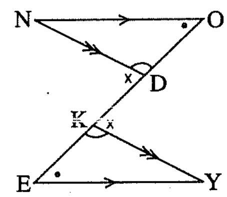
13.



Given: A is centre,
$\angle CAT = \angle MAT$
Show: Δ CAP $\cong$ Δ MAP

statement	reason
A is the centre	*given*
CA = MA	*radii are =*
$\angle CAT = \angle MAT$	*given*
$\angle CAP = \angle MAP$	*supp of = $\angle$s are =*
PA = PA	*same side*
ΔCAP $\cong$ ΔMAP	*SAS*

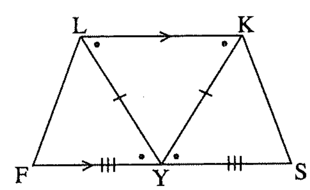
14.



Given: NO || EY,
ND || KY,
DO = KE
Show: Δ DON $\cong$ Δ KEY

statement	reason
NO || EY, ND || KY	*given*
$\angle NDK = \angle YKD$	*alt int $\angle$s*
$\angle NDO = \angle YKE$	*supp of = $\angle$s are =*
DO = KE	*given*
$\angle NOD = \angle YEK$	*alt int $\angle$s*
ΔDON $\cong$ ΔKEY	*ASA*

15.



Given: LK || FS,
LY = KY,
FY = YS
Show: Δ FLY $\cong$ Δ SKY

statement	reason
LY = KY	*given*
$\angle YLK = \angle YKL$	*$\angle$s opp = sides are =*
LK || FS	*given*
$\angle FYL = \angle YLK$	*alt int $\angle$s*
$\angle SYK = \angle YKL$	*alt int $\angle$s*
$\angle FYL = \angle SYK$	*both = to = $\angle$s*
FY = SY	*given*
Δ FLY $\cong$ Δ SKY	*SAS*

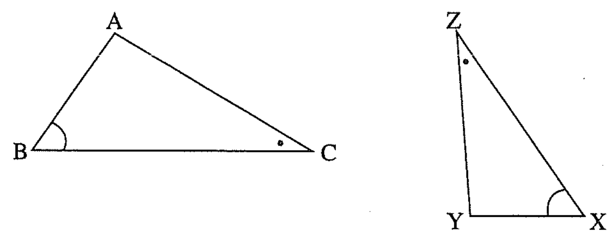
I.L.O. 9.14, 9.15

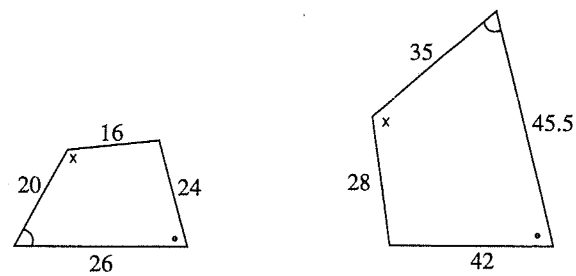
REVIEW
Find the missing term in a proportion.
Property of Pythagoras.

NEW
Similar polygons
Two polygons are similar if:
– corresponding angles are equal,
– corresponding sides are in proportion.

See the note about similar **polygons** on the next page.

Determine whether the two polygons are similar.





Corresponding ∠s are equal. Therefore:

Δ ABC ~ Δ YXZ

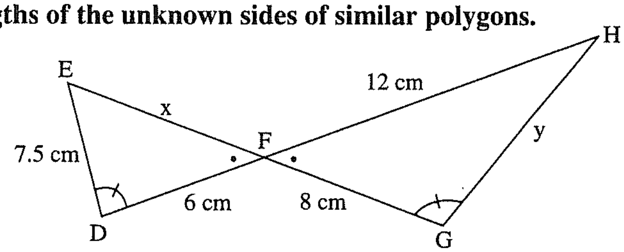
$\frac{AB}{YX} = \frac{BC}{XZ} = \frac{AC}{YZ}$ The order of the letters shows the correspondence of the vertices and sides.

Compare the ratios of corresponding sides. Start with the shortest side in each polygon.

$\frac{16}{28} = \frac{4}{7}$ $\frac{20}{35} = \frac{4}{7}$ $\frac{24}{42} = \frac{4}{7}$ $\frac{26}{45.5} = \frac{4}{7}$

Corresponding sides are in proportion. Therefore:
Quadrilateral JKLM ~ Quadrilateral RQPS

Find the lengths of the unknown sides of similar polygons.



Δ DEF ~ Δ GHF

$\frac{DE}{GH} = \frac{EF}{HF} = \frac{DF}{GF}$

$\frac{7.5}{y} = \frac{x}{12} = \frac{6}{8}$

$\frac{7.5}{y} = \frac{6}{8}$ $\frac{x}{12} = \frac{6}{8}$

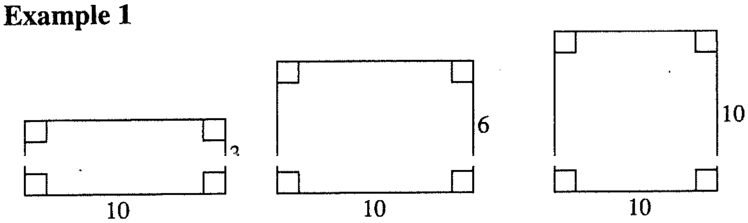
y = 10 cm x = 9 cm

Applications of similar polygons

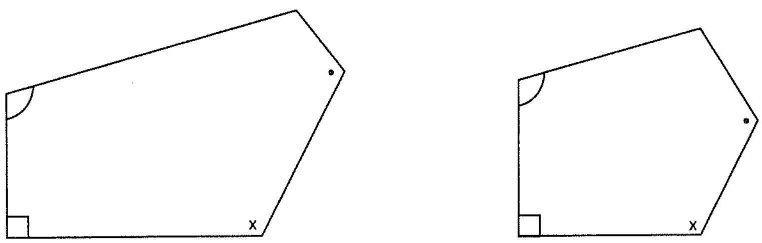
For application examples and questions, refer to the prescribed Math 9 textbook.

Note: Other than triangles, polygons with corresponding angles equal are not necessarily similar.

Example 1

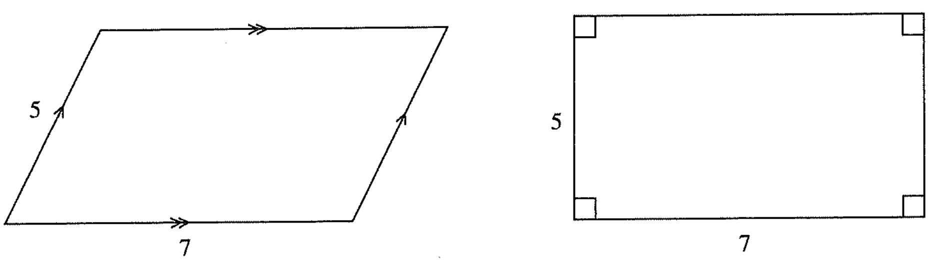


Example 2



Note: Other than triangles, polygons with corresponding sides in proportion are not necessarily similar.

Example



TEACHING ACTIVITIES

Similar polygons

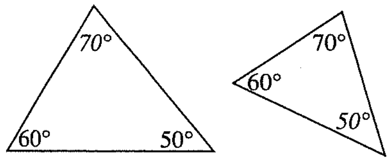
1. Have students make an enlargement of a polygon or a scale diagram of their bedrooms.

2. Have students draw two quadrilaterals that are equiangular but do not have corresponding sides in proportion. Discuss their results.

SIMILAR POLYGONS: ANSWERS

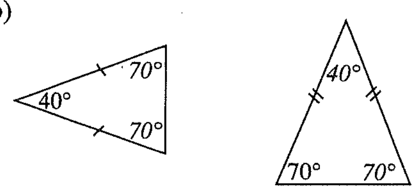
1. State why the two polygons are or are not similar.

a)



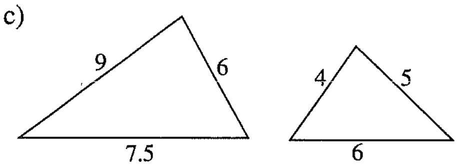
Yes, AAA

b)



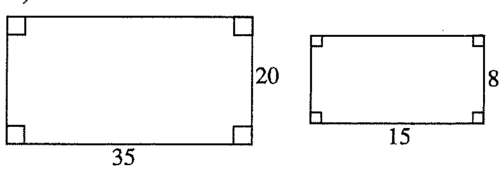
Yes, AAA

c)



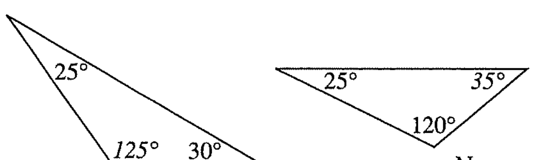
Yes, same ratios

d)



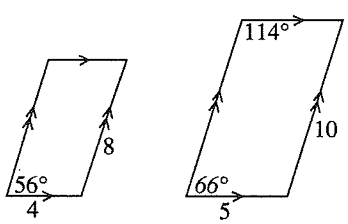
No, diff ratios

e)



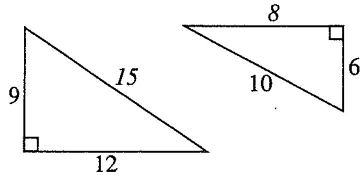
No, diff angles

f)



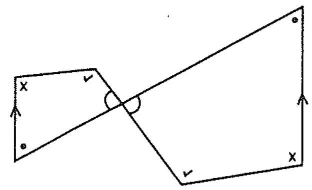
No, diff angles

g)



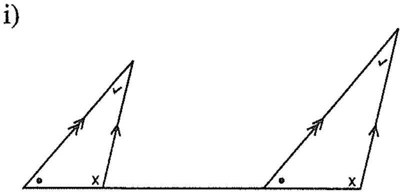
Yes, same ratios

h)



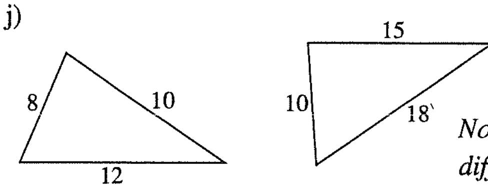
No, no information on ratios of sides although angles are equal

i)



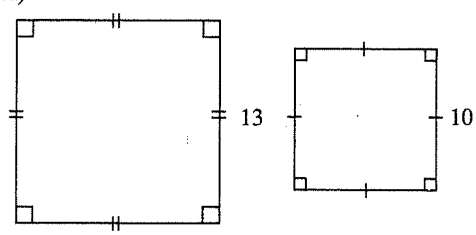
Yes, AAA

j)



No, diff ratios

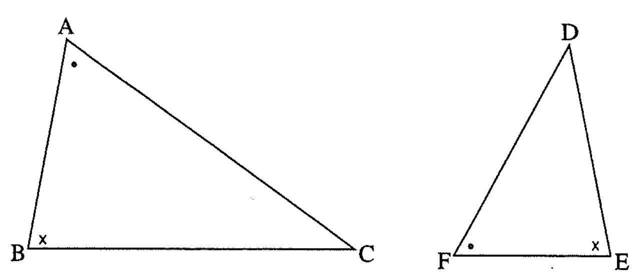
k)



Yes, equal angles and same ratios for sides

2. Complete each statement for the following pairs of similar figures.

a)

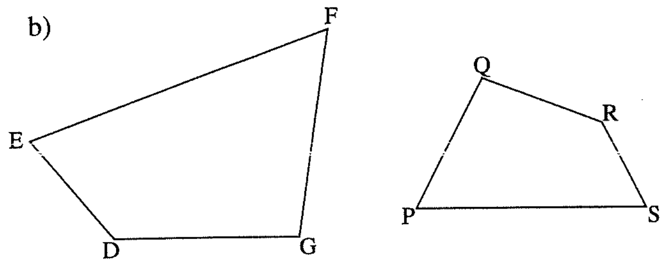


Δ ABC ~ Δ *FED*

$\frac{AB}{?} = \frac{BC}{?} = \frac{AC}{?}$

FE ED FD

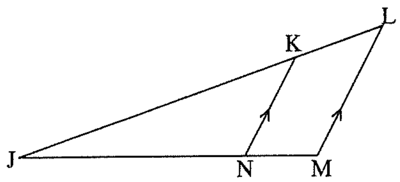
b)



DEFG ~ *RSPQ*

$$\frac{?\ FG}{PQ} = \frac{?\ GD}{QR} = \frac{?\ DE}{RS} = \frac{?\ FE}{PS}$$

c)



Δ JKN ~ Δ *JLM*

$$\frac{JK}{?\ JL} = \frac{?\ KN}{LM} = \frac{?\ JN}{?\ JM}$$

3. Complete the following statement.

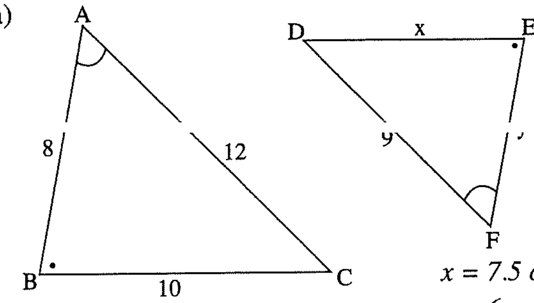
If ΔDHG ~ ΔMPT, then

$$\frac{?\ HG}{PT} = \frac{DG}{?\ MT} = \frac{?\ DH}{?\ MP}$$

4. Use the ratios of the corresponding sides to calculate the unknown lengths in the following similar figures. (All measurements are in cm.)

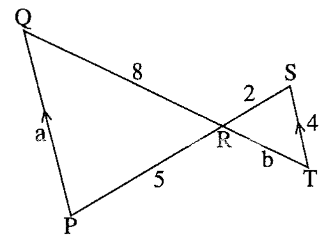
Work in your notebook.

a)



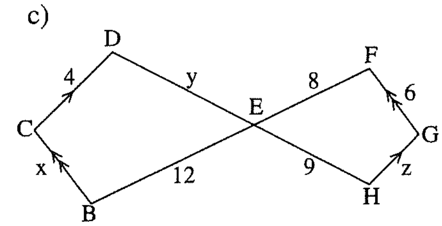
x = 7.5 cm
y = 6 cm

b)



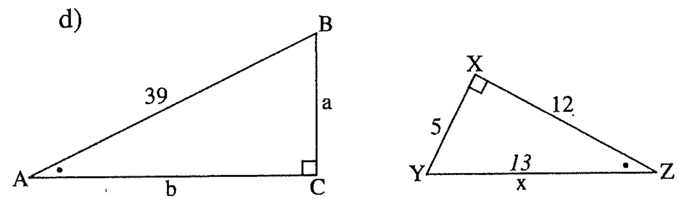
a = 10 cm
b = 3.2 cm

c)



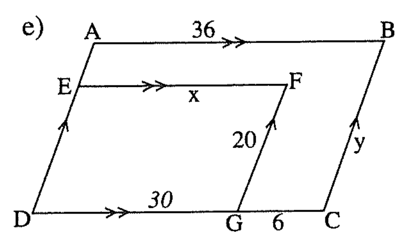
x = 9 cm
y = 13.5 cm
z = $2.\dot{6}$ cm

d)



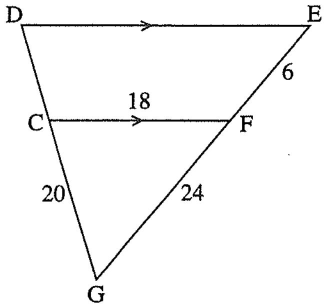
x = 13 cm
a = 15 cm
b = 36 cm

e)



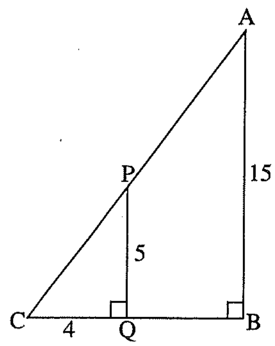
x = 30 cm
y = 24 cm

f)



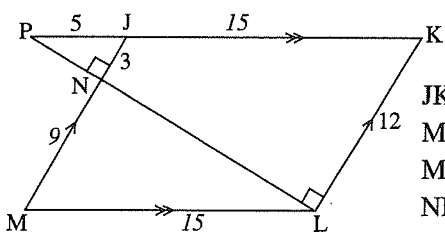
DE = ? *22.5 cm*
DG = ? *25 cm*
DC = ? *5 cm*

g)



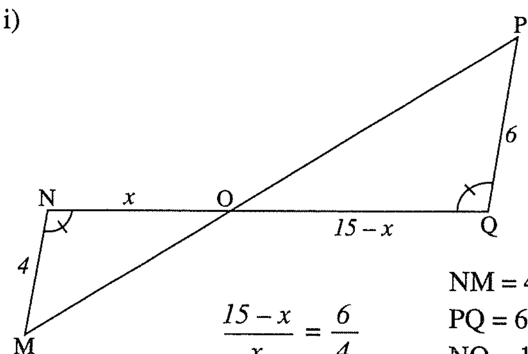
CB = ? *12*
QB = ? *8*
CP = ? $\sqrt{41}$

h)



JK = ? *15*
ML = ? *15*
MN = ? *9*
NL = ? *12*

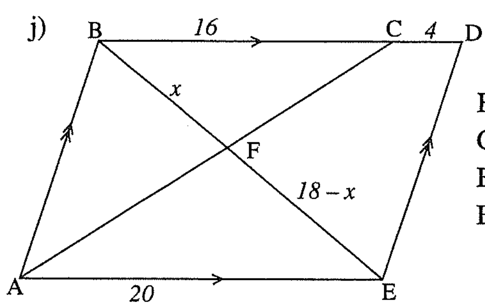
i)



$$\frac{15-x}{x} = \frac{6}{4}$$

NM = 4
PQ = 6
NQ = 15
NO = ? *6*

j)



BC = 16
CD = 4
BE = 18
BF = ?*8*

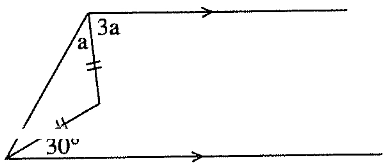
$$\frac{18-x}{20} = \frac{x}{16}$$

PROBLEMS: ANSWERS

Note: Detailed answers to questions 1 to 11 appear on pages T43 and T44.

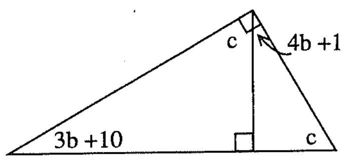
1.

a)



a = *30°*

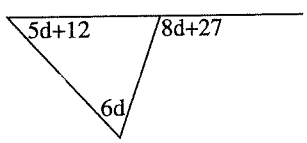
b)



b = *9°*

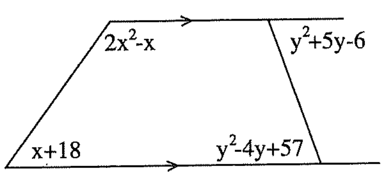
c = *53°*

c)



d = *5°*

d)

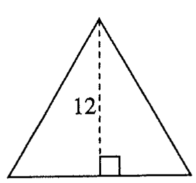


x = *±9°*

y = *7°*

2. The legs of a right triangle are 7 cm and 24 cm long. Find the length of each side of a similar triangle if its perimeter is 224 cm. *28, 96, 100 cm*

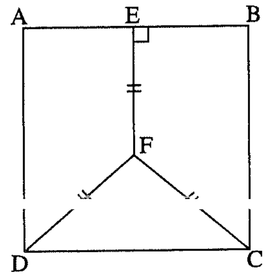
3.



If the perimeter of an isosceles triangle is 36 cm and the height to the base is 12 cm, what is the area of the triangle? *60 cm²*

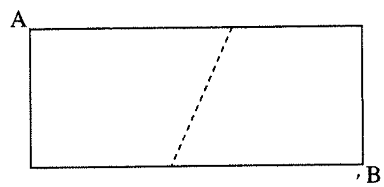
4. A triangle has sides of lengths 29 cm, 29 cm, and 40 cm. Find another isosceles triangle with the same perimeter and area that also has sides of integral lengths. *37, 37, 24 cm*

5.



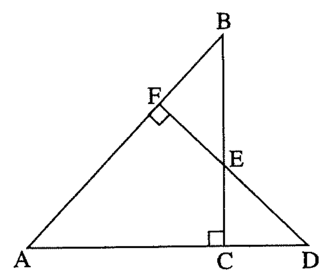
ABCD is a square with sides of 16 cm. FE is perpendicular to AB and FE = FD = FC. Find the length of FC. *10 cm*

6.



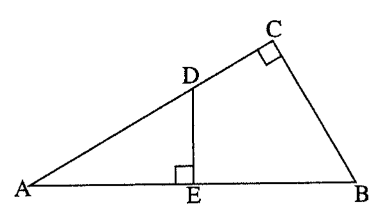
A paper rectangle is folded so that B falls on A. If the rectangle is 6 cm by 8 cm, find the length of the fold. *7.5 cm*

7.



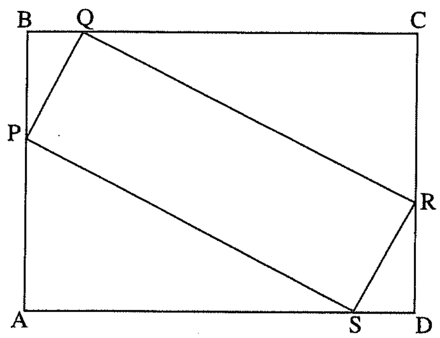
Two isosceles right triangles with legs 5 cm long overlap as shown. Find the area of quadrilateral AFEC. *10.4 cm²*

8.



If AE = 6 cm, EB = 7 cm, and BC = 5 cm, find the area of quadrilateral BCDE. *22.5 cm²*

9.



ABCD and PQRS are rectangles. AP = 6 cm, SD = 2 cm, and RD = 3 cm. Find the perimeter of ABCD. *40 cm*

10. In quadrilateral ABCD, $\angle A = 120°$. Find the other three angles if $\angle B$ and $\angle C$ are complementary and $\angle C$ and $\angle D$ are supplementary. $\angle B = 60°$, $\angle C = 30°$, $\angle D = 150°$

11. The four angles of a quadrilateral are consecutive odd integers. Find the largest angle. *93°*

ANSWERS

1.

a) $3a + a + a + 30 = 180°$
$5a = 150°$
$a = 30°$ a = $30°$

b) $4b + 1 = 3b + 10$ b = $9°$
$b = 9$
$c = 90 - 37$ c = $53°$

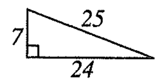
c) $180 - (5d + 12 + 6d) + 8d + 27 = 180°$
$-3d + 15 = 0$
$-3d = -15$ d = $5°$
$d = 5$

d) $2x^2 - x + x + 18 = 180$
$2x^2 = 162$ x = $\pm 9°$
$x^2 = 81$
$x = \pm 9$ y = $7°$

Check angles, $2(9)^2 - 9 = 153°$
$9 + 18 = 27$
$2(-9)^2 - 9 = 171°$
$-9 + 18 = 9$

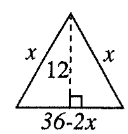
$y^2 + 5y - 6 = y^2 - 4y + 57$
$9y = 63$
$y = 7$

2.



The perimeter is 56 cm. Similar Δ is 4 times as big.
Sides are 28, 96, 100 cm.

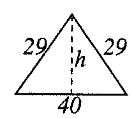
3.



$12^2 + (18 - x)^2 = x^2$
$144 + 324 - 36x + x^2 = x^2$
$468 = 36x$
$13 = x$

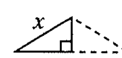
$\text{area} = \frac{1}{2} \cdot 10 \cdot 12 = 60$ *60 cm²*

4.



$h^2 + 20^2 = 29^2$
$h^2 + 400 = 841$
$h^2 = 441$
$h = \pm 21$

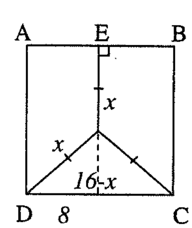
$\text{area} = \frac{1}{2} \cdot 40 \cdot 21 = 420 \text{ cm}^2$

Base	Height		Perimeter
factors of 840			
84	10	$\sqrt{42^2 + 10^2} \doteq 41$	$\doteq$ 160 too big
42	20	$\sqrt{21^2 + 20^2} = 42$	126 too big
35	24	$\sqrt{17.5^2 + 24^2} \doteq 30$	$\doteq$ 95 close
24	35	$\sqrt{12^2 + 35^2} = 37$	98 got it

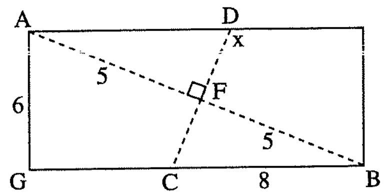
37, 37, 24 cm

5.

$8^2 + (16 - x)^2 = x^2$
$64 + 256 - 32x + x^2 = x^2$
$320 = 32x$
$10 = x$ *10 cm*



6. *CD is the perp bisector of AB.*
$\Delta ADF \sim \Delta BAG$



$$\frac{x}{6} = \frac{5}{8}$$

$$x = \frac{30}{8}$$

$$CD = 2 \cdot \frac{30}{8} = \frac{30}{4}$$

7.5 cm

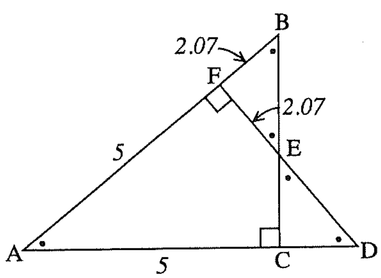
7. *In rt* ΔACB, $AB = \sqrt{5^2 + 5^2} = 7.07$
all marked angles = 45°

$$\text{area } \Delta ACB = \frac{1}{2} \cdot 5 \cdot 5 = 12.5$$

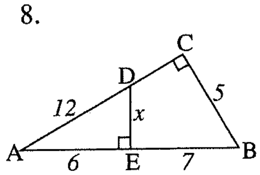
$$\text{area } \Delta EFB = \frac{1}{2}(2.07)(2.07) = 2.14$$

area quad ACEF = 10.36

10.4 cm^2



8. $AC = \sqrt{13^2 - 5^2} = 12$
$\Delta AED \sim \Delta ACB$



$$\frac{x}{6} = \frac{5}{12}$$

$$x = 2.5$$

$$\text{Area } \Delta ABC = \frac{1}{2} \cdot 12 \cdot 5 = 30$$

$$\text{Area } \Delta AED = \frac{1}{2} \cdot 6 \cdot 2.5 = 7.5$$

Area quad BCDE = 22.5

22.5 cm^2

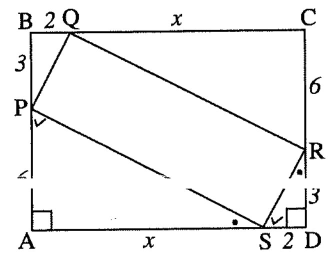
9. $\Delta SAP \sim \Delta RDS$

$$\frac{x}{6} = \frac{3}{2}$$

$$x = 9$$

perimeter $= 2(11 + 9)$

40 cm



10.

A	B	C	D
120°	$90 - x$	x	$180 - x$

$\overset{+90°}{\frown}$ (B to C), $\overset{+180°}{\frown}$ (C to D)

$$120 + 90 - x + x + 180 - x = 360$$

$$x = 30$$

$\angle B = 60°$, $\angle C = 30°$, $\angle D = 150°$

11.

1st	2nd	3rd	4th
n	$n + 2$	$n + 4$	$n + 6$

$$n + n + 2 + n + 4 + n + 6 = 360$$

$$4n + 12 = 360$$

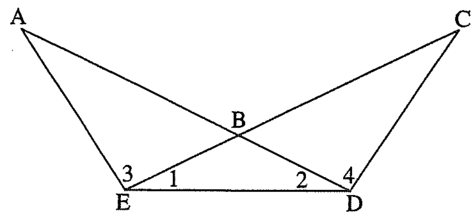
$$4n = 348$$

$$n = 87$$

87°, 89°, 91°, 93°

OVERLAPPING TRIANGLES: ANSWERS

1.

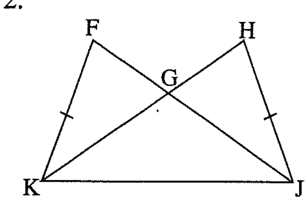


Identify the two congruent triangles in the diagram for which the following are corresponding parts.

a) $\angle 1 = \angle 2$	*ΔCED, ΔADE*
b) $\angle 3 = \angle 4$	*ΔAEB, ΔCDB*
c) $\angle AED = \angle CDE$	*ΔAED, ΔCDE*
d) $AB = CB$	*ΔABE, ΔCBD*
e) $AD = EC$	*ΔADE, ΔCED*
f) $AE = CD$	*ΔAEB, ΔCDB or ΔAED, ΔCDE*

Complete each of the following proofs.

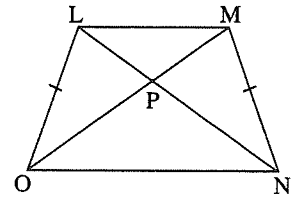
2.



Given: $FK = HJ$,
$FJ = HK$
Show: $\Delta FJK \cong \Delta HKJ$

statement	**reason**
$FK = HJ$	*given*
$FJ = HK$	*given*
$KJ = KJ$	*same side*
$\Delta FJK \cong \Delta HKJ$	*SSS*

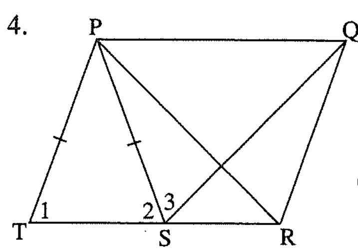
3.



Given: $LO = MN$,
$\angle LON = \angle MNO$
Show: $\Delta LON \cong \Delta MNO$

statement	**reason**
$LO = MO$	*given*
$\angle LON = \angle MNO$	*given*
$ON = ON$	*same side*
$\Delta LON \cong \Delta MNO$	*SAS*

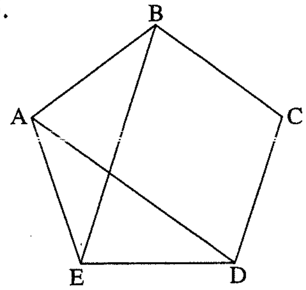
4.



Given: $\angle 1 = \angle 2 = \angle 3$,
$TR = SQ$
Show: $\Delta PTR \cong \Delta PSQ$

statement	**reason**
$\angle 1 = \angle 2$	*given*
$PT = PS$	*sides opp = $\angle$s are =*
$\angle 1 = \angle 3$	*given*
$TR = SQ$	*given*
$\Delta PTR \cong \Delta PSQ$	*SAS*

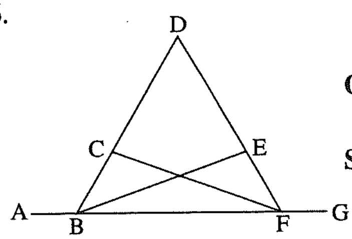
5.



Given: ABCDE is a regular pentagon
Show: $\Delta ABE \cong \Delta EAD$

statement	reason
ABCDE is a regular pentagon	*given*
BA = AE	*defn of reg pentagon*
$\angle BAE = \angle AED$	*defn of reg pentagon*
AE = ED	*defn of reg pentagon*
$\Delta ABE \cong \Delta EAD$	*SAS*

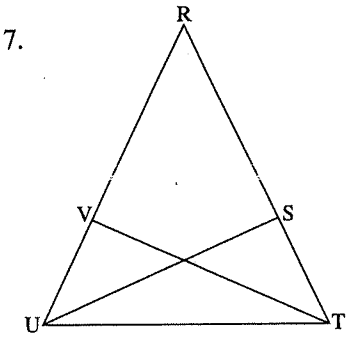
6.



Given: $\angle ABD = \angle GFD$, CD = DE
Show: $\Delta DEB \cong \Delta DCF$

statement	reason
$\angle ABD = \angle GFD$	*given*
$\angle DBF = \angle DFB$	*supp of = $\angle$s are =*
DB = DF	*sides opp = $\angle$s are =*
$\angle D = \angle D$	*same $\angle$*
CD = DE	*given*
$\Delta DEB \cong \Delta DCF$	*SAS*

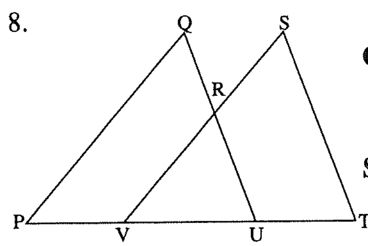
7.



Given: RU = RT, $TV \perp RU$, $US \perp RT$
Show: $\Delta UVT \cong \Delta TSU$

statement	reason
$TV \perp RU$, $US \perp RT$	*given*
$\angle UVT = \angle TSU = 90°$	*defn of $\perp$*
RU = RT	*given*
$\angle RUT = \angle RTU$	*$\angle$s opp = sides are =*
$\angle VTU = \angle SUT$	*3rd $\angle$s of Δs are =*
UT = UT	*same side*
$\Delta UVT \cong \Delta TSU$	*ASA*

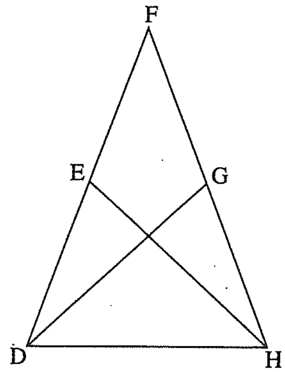
8.



Given: PV = UT, PQ || VS, QU || ST
Show: $\Delta QPU \cong \Delta SVT$

statement	reason
PQ || VS, QU || ST	*given*
$\angle QPU = \angle SVT$	*corr $\angle$s*
$\angle QUP = \angle STV$	*corr $\angle$s*
PV = UT	*given*
VU = VU	*same segment*
PV + VU = VU + UT	equation prop of addition
PU = VT	substitution
$\Delta QPU \cong \Delta SVT$	*ASA*

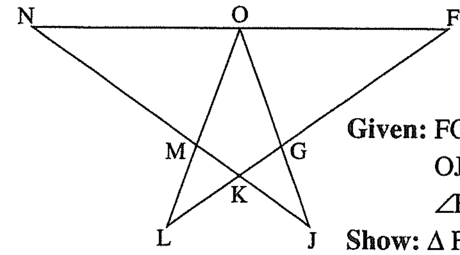
9.



Given: FD = FH,
FE = FG
Show: Δ DEH ≅ Δ HGD

statement	reason
FD = FH	*given*
∠FDH = ∠FHD	*∠s opp = sides are =*
FE = FG	*given*
DF – EF = FH – FG	*eq prop of subtract*
DE = HG	*substitution*
DH = DH	*same side*
Δ DEH ≅ Δ HGD	*SAS*

10.



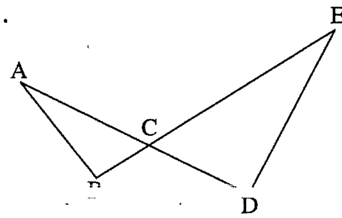
Given: FO = ON,
OJ = OL,
∠FOJ = ∠NOL
Show: Δ FOL ≅ Δ NOJ

statement	reason
FO = ON	*given*
∠FOJ = ∠NOL	*given*
∠JOL = ∠JOL	*same angle*
∠FOJ + ∠JOL = ∠JOL + ∠NOL	*eq prop of addition*
∠FOL = ∠NOJ	*substitution*
OL = OJ	*given*
Δ FOL ≅ Δ NOJ	*SAS*

SIMILAR TRIANGLES: ANSWERS

Complete each of the following proofs.

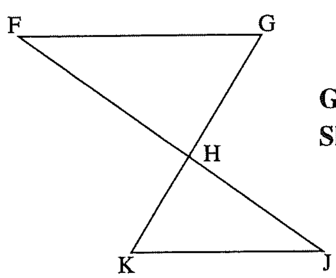
1.



Given: $\angle B = \angle D$
Show: $\Delta ABC \sim \Delta EDC$

statement	reason
$\angle B = \angle D$	*given*
$\angle ACB = \angle ECD$	*vert opp ∠s*
$\angle A = \angle E$	*3rd ∠s of Δ s are =*
$\Delta ABC \sim \Delta EDC$	AAA

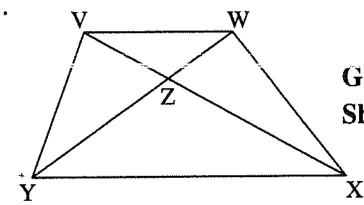
2.



Given: $FG \parallel KJ$
Show: $\frac{FG}{JK} = \frac{GH}{KH}$

statement	reason
$FG \parallel KJ$	*given*
$\angle F = \angle J$	*alt int ∠s*
$\angle G = \angle K$	*alt int ∠s*
$\angle FHG = \angle JHK$	*vert opp ∠s*
$\Delta FHG \sim \Delta JHK$	*AAA*
$\frac{FG}{JK} = \frac{GH}{KH}$	corresponding sides of similar figures are in proportion

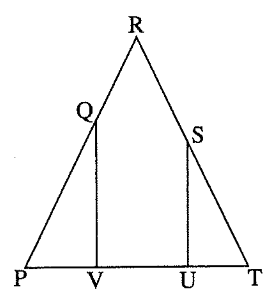
3.



Given: $VW \parallel YX$
Show: $\Delta WVZ \sim \Delta YXZ$

statement	reason
$VW \parallel YX$	*given*
$\angle WVZ = \angle ZXY$	*alt int ∠s*
$\angle VWZ = \angle ZYX$	*alt int ∠s*
$\angle VZW = \angle YZX$	*vert opp ∠s*
$\Delta WVZ \sim \Delta YXZ$	*AAA*

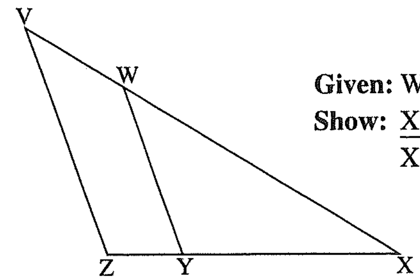
4.



Given: $RP = RT$,
$QV \perp PT$,
$SU \perp PT$
Show: $\frac{QV}{SU} = \frac{PV}{TU}$

statement	reason
$RP = RT$	*given*
$\angle P = \angle T$	*∠s opp = sides are =*
$QV \perp PT$, $SU \perp PT$	*given*
$\angle PVQ = \angle SUT = 90°$	*defn of ⊥*
$\angle PQV = \angle TSU$	*3rd ∠s of Δs are =*
$\Delta PQV \sim \Delta TSU$	*AAA*
$\frac{QV}{SU} = \frac{PV}{TU}$	*corr sides of sim Δ s are in proportion*

5.

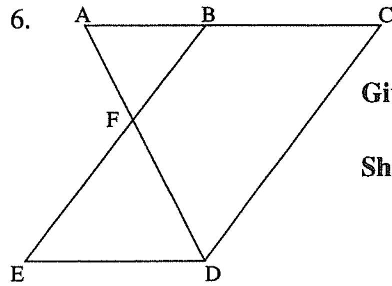


Given: WY | | VZ

Show: $\frac{XW}{XV} = \frac{XY}{XZ}$

statement	reason
WY | | VZ	*given*
$\angle XWY = \angle XVZ$	*corr ∠s*
$\angle XYW = \angle XZV$	*corr ∠s*
$\angle X = \angle X$	*same ∠*
$\Delta XWY \sim \Delta XVZ$	*AAA*
$\frac{XW}{XV} = \frac{XY}{XZ}$	*corr sides of sim Δ s are in proportion*

6.

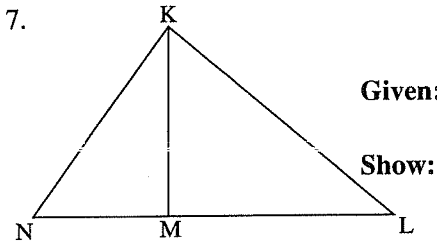


Given: BCDE is a parallelogram

Show: $\frac{FE}{DC} = \frac{DE}{AC}$

statement	reason
BCDE is a parallelogram	*given*
AC | | ED, BE | | CD	*defn of / / gram*
$\angle FDE = \angle FAB$	*alt int ∠s*
$\angle E = \angle C$	*opp ∠s of / / gram*
$\angle DFE = \angle ADC$	*3rd ∠s of Δs are =*
$\Delta DFE \sim \Delta ADC$	*AAA*
$\frac{FE}{DC} = \frac{DE}{AC}$	*corr sides of sim Δs are in proportion*

7.

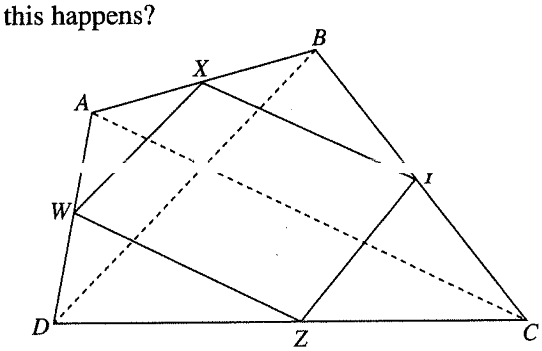


Given: $\angle NKL = 90°$, $KM \perp NL$
Show: $\Delta KMN \sim \Delta LKN$

statement	**reason**
$KM \perp NL$	*given*
$\angle KMN = 90°$	*defn of* $\perp$
$\angle NKL = 90°$	*given*
$\angle KMN = \angle NKL$	*both = 90°*
$\angle N = \angle N$	*same* $\angle$
$\angle NKM = \angle KLN$	*3rd* $\angle$*s of* Δ*s are =*
$\Delta KMN \sim \Delta LKN$	*AAA*

8. On a sheet of paper, draw 3 fairly large, differently shaped triangles. On each, draw a segment joining the midpoints of two of the sides. What observations can you make about the segment and the third side of the triangle? Can you explain why this happens?
The segment is $\parallel$ *to the 3rd side.*
The segment equals 1/2 the 3rd side.
The two overlapping Δ*s are similar and the sides are in the ratio 2:1.*

9. On a sheet of paper, draw 3 fairly large, differently shaped quadrilaterals. (One should be concave.) On each, join the midpoints of consecutive sides to form a new quadrilateral. What observations can you make about the new quadrilateral? Can you explain why this happens?



XY and WZ are parallel to AC, and YZ and XW are parallel to BD. Therefore, $XY \parallel WZ$, $WX \parallel ZY$.
The new quadrilateral is a parallelogram.

PERIMETER, AREA, AND VOLUME OF SIMILAR FIGURES: ANSWERS

Objective: To discover the relationship between the lengths of corresponding sides and the perimeters, areas, and volumes of similar figures.

Complete the following tables. (All units are in cm.)

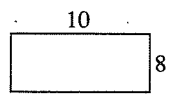
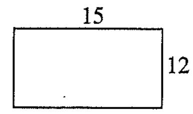
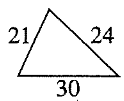
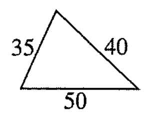
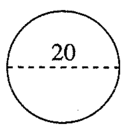
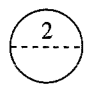
Figure	Perimeter	Similar figure	Perimeter	Ratio of corresponding sides	Ratio of perimeters
	36 cm		54 cm	$\frac{15}{10}=\frac{3}{2}$	$\frac{54}{36}=\frac{3}{2}$
	*75 cm*		*125 cm*	$\frac{50}{30}=\frac{5}{3}$	$\frac{125}{75}=\frac{5}{3}$
	*62.8 cm*		*6.28 cm*	$\frac{2}{20}=\frac{1}{10}$	$\frac{6.28}{62.8}=\frac{1}{10}$

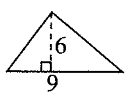
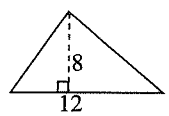
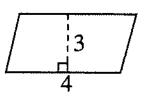
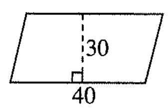
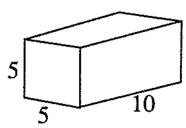
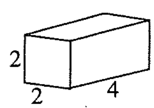
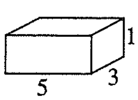
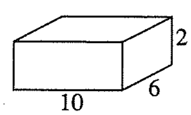
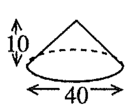
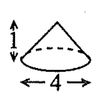
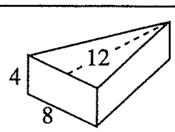
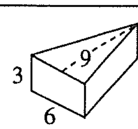
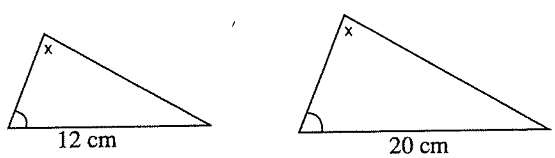
Figure	Area	Similar figure	Area	Ratio of corresponding sides	Ratio of areas
	*27 cm²*		*48 cm²*	$\frac{12}{9}=\frac{4}{3}$	$\frac{48}{27}=\frac{16}{9}$
	*12 cm²*		*1200 cm²*	$\frac{40}{4}=\frac{10}{1}$	$\frac{1200}{12}=\frac{100}{1}$
	surface area *250 cm²*		surface area *40 cm²*	$\frac{4}{10}=\frac{2}{5}$	$\frac{40}{250}=\frac{4}{25}$

Figure	Volume	Similar figure	Volume	Ratio of corresponding sides	Ratio of volumes
	*15 cm³*		*120 cm³*	$\frac{10}{5}=\frac{2}{1}$	$\frac{120}{15}=\frac{8}{1}$
	$4186.\dot{6}$ *cm³*		$4.18\dot{6}$ *cm³*	$\frac{4}{40}=\frac{1}{10}$	$\frac{4.18\dot{6}}{4186.\dot{6}}=\frac{1}{1000}$
	*192 cm³*		*81 cm³*	$\frac{6}{8}=\frac{3}{4}$	$\frac{81}{192}=\frac{27}{64}$

1. Complete the following table to show the relationship between the corresponding sides, perimeters, areas, and volumes of two similar figures.

Ratio of sides	Ratio of perimeters	Ratio of areas	Ratio of volumes
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{8}{27}$
$\frac{10}{1}$	$\frac{10}{1}$	$\frac{100}{1}$	$\frac{1000}{1}$
$\frac{5}{4}$	$\frac{5}{4}$	$\frac{25}{16}$	$\frac{125}{64}$
$\frac{3}{10}$	$\frac{3}{10}$	$\frac{9}{100}$	$\frac{27}{100}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{a}{b}$	$\frac{a}{b}$	$\frac{a^2}{b^2}$	$\frac{a^3}{b^3}$

2.

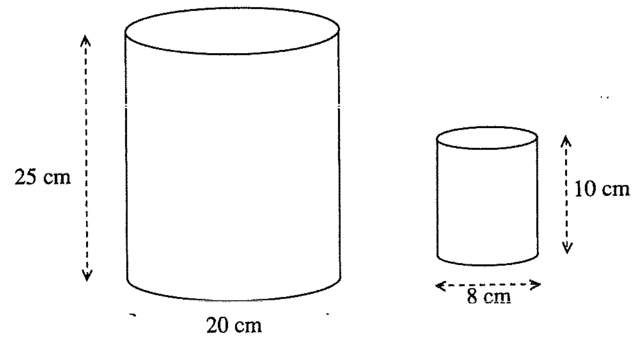


For the two triangles shown above, find in simplest form the ratio of:

a) the corresponding sides $\frac{20}{12} = \frac{5}{3}$

b) the area of the triangles $\frac{25}{9}$

3.



For the two cylinders shown above, find in simplest form the ratio of:

a) the heights $\frac{10}{25} = \frac{2}{5}$

b) the surface areas $\frac{4}{25}$

c) the volumes $\frac{8}{125}$

4. If it takes 4 minutes to cross-country ski around a soccer pitch 120 m by 60 m, how long will it take to ski around a similar field 300 m by 150 m? *10 min*

Ratio of sides: $\frac{300}{120} = \frac{5}{2}$

Ratio of perimeters: $\frac{5}{2}$

Time: $\frac{5}{2} \bullet 4 = 10$

5. A model of a yacht is made to a scale of 1:20. If the sails on the model use 0.1 m^2 of cloth, how much will the sails for the yacht need?
40 m^2; area 1:400

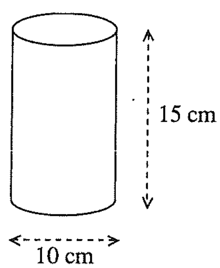
6. A daffodil bed in the park is planted with 200 bulbs. A similarly shaped bed is planted with 1800 bulbs. How much larger is one bed than the other?
3 times
area 1:9
sides 1:3

7. Two square paddocks are fenced to graze 4 zebra and 36 zebra respectively. If the smaller paddock takes 120 m of fencing, how much will the larger need? *360 m*
area 1:9
perimeter 1:3

8. Clams vary in size from as small as a pinhead to as large as 1.3 m in length. If a clam 5 cm long weighs 25 g, what would you expect a similar clam, 1 m long, to weigh? *200 kg*
length 1:20
vol 1:8000

9. A ball of string 10 cm in diameter costs $1.60. What would you expect a similar ball of string, 15 cm in diameter, to cost? *$5.40*
diam 2:3
vol 8:27

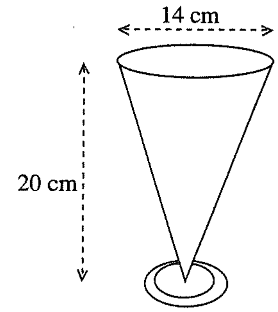
10.



vol 1:1000
side 1:10

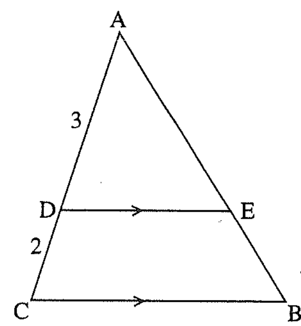
A litre of oil is sold in cans as shown. Large drums of a similar shape hold 1 kL of oil. What are the dimensions of the drum?
diameter 100 cm; height 150 cm

11.



The glass shown holds 1 kg of liquid when full. To what depth will 125 g of liquid fill the glass?
10 cm
vol 1:8
height 1:2

12.



Find the ratio of:

a) $\frac{\text{area } \Delta \text{ ADE}}{\text{area } \Delta \text{ ABC}}$ $\frac{3^2}{5^2} = \frac{9}{25}$

b) $\frac{\text{area } \Delta \text{ ADE}}{\text{area BCDE}}$ $\frac{9}{16}$

c) $\frac{\text{area BCDE}}{\text{area } \Delta \text{ ABC}}$ $\frac{16}{25}$