Pre-Calculus 12 **5.4-5.8 Self-Assessment** **Name:** Key

A1. Write $\log_a(b+2)=c$ in exponential form.

$b+2=a^c$

A2. Write $y-1=3^{x+2}$ in logarithmic form.

$\log_3(y-1)=x+2$

B1. Evaluate: $\log_{\sqrt{5}} 125 = x$

$125=\sqrt{5}^{\,x} \rightarrow 125=5^{\frac{1}{2}x} \rightarrow 5^3=5^{\frac{1}{2}x} \rightarrow 3=\frac{x}{2} \rightarrow x=6$

B2. Evaluate: $2\log_4 16+\frac{1}{3}\log_2\left(\frac{1}{8}\right)$

$2\log_4 4^2+\frac{1}{3}\log_2 2^{-3} \rightarrow 2(2)+\frac{1}{3}(-3) \rightarrow 4-1=3$

C. Use benchmarks to estimate the value of $\log_2 60$.

$\log_2 32 \rightarrow \log_2 2^5$

$\log_2 64 \rightarrow \log_2 2^6$

$\log_2 60 \approx 5.9$

D1. Write $3\log a+\frac{1}{2}\log b-\frac{1}{4}\log c$ as a single logarithm.

$\log a^3+\log b^{1/2}-\log c^{1/4} \rightarrow \log\frac{a^3\sqrt{b}}{\sqrt[4]{c}}$

D2. Evaluate $\log_3\sqrt{54}-\log_3\sqrt{6}$.

$\log_3\frac{\sqrt{54}}{\sqrt{6}}=\log_3\sqrt{9}=\log_3 3=1$

D3. Write $\log\left(\frac{a^2}{bc^3}\right)$ in terms of $\log a$, $\log b$, and/or $\log c$.

$2\log a-\log b-3\log c$

D4. If $\log_3 x=2$ and $\log_3 y=5$, evaluate $\log_3\left(\frac{3x^2}{y}\right)$.

$\log_3 3+2\log_3 x-\log_3 y$

$=1+2(2)-5$

$=1+4-5=0$

E. Evaluate to 3 decimal places: $\log_4 75$.

$\frac{\log 75}{\log 4}=3.114$

F. Sketch the graph of $y=\log_2(x+2)+1$.

$2^y=x$

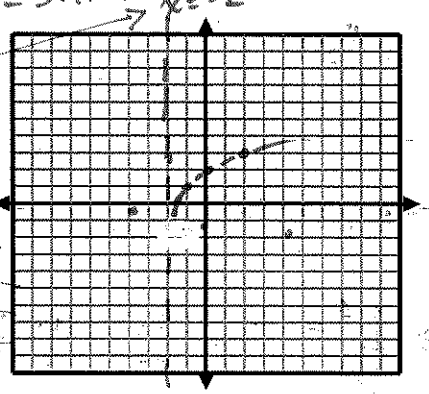
x	y
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4

$y=\log_2 x$

x	y
$1/4$	-2
$1/2$	-1
1	0
2	1
4	2

$y=\log_2(x+2)+1$

$-3/4$	-1
$-3/2$	0
-1	1
0	2
2	3



G. Determine the domain, range, equation of the asymptote, and intercepts of the graph in F.

$x=-2$

y-int $=(0,2)$

x-int $=(-3/2,0)$

D: $x>-2$

R: $y\in\mathbb{R}$

H1. Solve for x to 3 decimal places: $5^{x-3} = 2^{x+1}$.

$\log 5^{x-3} = \log 2^{x+1}$

$(x-3)\log 5 = (x+1)\log 2$

$x\log 5 - 3\log 5 = x\log 2 + \log 2$

$x\log 5 - x\log 2 = 3\log 5 + \log 2$

$x(\log 5 - \log 2) = 3\log 5 + \log 2$

$x = \dfrac{\log 5^3 \cdot 2}{\log 5/2} = \dfrac{\log 250}{\log 2.5}$

H2. Solve for x to 3 decimal places: $2^x = 3(4^{x+1})$.

$\log 2^x = \log 3(4^{x+1})$

$x\log 2 = \log 3 + \log 4^{x+1}$

$x\log 2 = \log 3 + (x+1)\log 4$

$x\log 2 = \log 3 + x\log 4 + \log 4$

$x\log 2 - x\log 4 = \log 3 + \log 4$

$x(\log 2 - \log 4) = \log(3\cdot 4)$

$x = \dfrac{\log(3.4)}{\log 1/4} = \dfrac{\log 12}{\log(1/4)} = -3.58$

$= \log 6.02/6$

I1. Solve for x: $\log(x+11) + \log x = \log(x+1) + \log 6$.

$\log x(x+11) = \log 6(x+1)$

$x^2 + 11x = 6x + 6$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1)$

$x = -6, 1$

I2. Solve for x: $\log_2(x+2) + \log_2 x = 3$.

$\log_2(x+2)x = \log_2 2^3$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = \cancel{-4}, 2$

$\cancel{-4}$ reject

J1. You invest \$5000 in an account with a fixed interest rate of 3%/annum, compounded semi-annually. How long will it take for the investment to double?

$10{,}000 = 5{,}000\left(1 + \dfrac{0.03}{2}\right)^{t(2)}$

$2 = 1.015^{2t}$

$\log 2 = 2t \log 1.015$

$\dfrac{\log 2}{2\log 1.015} = t$

$t = 23.3$ years

J2. Parents plan to invest money for their newborn son so that he has \$20 000 available for his education on his 18th birthday. Assuming a growth rate of 6% per year, compounded monthly, how much will they need to invest today?

$20{,}000 = P_0\left(1 + \dfrac{0.06}{12}\right)^{12(18)} \rightarrow P_0 = \dfrac{20{,}000}{(1.005)^{12(18)}} = \$6810.21$

J3. A radioactive isotope has a half-life of approximately 12 weeks. How much of a sample of 30 grams would remain after 50 weeks? (Round to the nearest hundredth of a gram)

$P = 30\left(\dfrac{1}{2}\right)^{\frac{50}{12}} = 1.67$ grams

K1. How many times as intense as a 6.3 magnitude earthquake is an 8.4 magnitude earthquake?

$\dfrac{10^{8.4}}{10^{6.3}} = 10^{8.4-6.3} = 10^{2.1} = 126$ times

K2. How many times louder is a referee's whistle (125 dB) than a flute (89 dB)?

$L = 10\log_{10}\left(\dfrac{I}{I_0}\right) \rightarrow \dfrac{L}{10} = \log_{10}\left(\dfrac{I}{I_0}\right) \rightarrow 10^{\frac{L}{10}} = \dfrac{I}{I_0} \rightarrow I_0 10^{\frac{L}{10}} = I$

$\dfrac{I_0 10^{12.5/10}}{I_0 10^{8.9/10}} = 10^{\frac{3.6}{10}}$

$= 3981 \times$

K3. Tomato juice has a pH level of 4.0 Determine the pH level of a solution that is 5 times more acidic.

$\dfrac{10^{-4}}{10^{-x}} = 5 \rightarrow 10^{4-x} = 5$

$4 - x = \log_{10} 5$

$4 - \log 5 = x$

$x = 3.3$

$pH = 3.3$